

Scheme Of Valuation

Scoring Indicators

Revision : 2015 Course Code:1002

Course Title : Engineering Mathematics -I

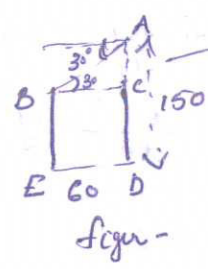
Qn.No	sub.divi sion	Scoring indicator	Split up Score	Sub Total	Total
I	1.	$\sin \theta = \frac{1}{2}, \cos \theta = \sqrt{1 - \sin^2 \theta} = \sqrt{1 - \frac{1}{4}} = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$	1+1		
	2.	Area of triangle $\Delta = \frac{1}{2} bc \sin A = \frac{1}{2} \times 3 \times 2 \times \sin 30$ $= \frac{1}{2} \times 3 \times 2 \times \frac{1}{2}$ $= \frac{3}{2} \text{ cm}^2$	1  1		
	3.	$\lim_{x \rightarrow 1} \frac{2x+3}{4x-1} = \frac{2 \times 1 + 3}{4 \times 1 - 1} = \frac{5}{3}$	1+1		
	4.	$y = 3 \cos x - 4 \tan x$ $\frac{dy}{dx} = -3 \sin x - 4 \sec^2 x$	1+1		
	5.	$V = a^3$ $\frac{dV}{da} = 3a^2$	1 1		
				10	



Qn.No	sub.division	Scoring indicator	Split up Score	Sub Total	Total
	5.	$y = \sin x.$ Let $\Delta x$ be small increment in $x$ , corresponding increment in $y$ be $\Delta y$ . $y + \Delta y = \sin(x + \Delta x)$ $\Delta y = \sin(x + \Delta x) - \sin x$ $= 2 \cos \frac{x + \Delta x + x}{2} \cdot \sin \frac{x + \Delta x - x}{2}$ $= 2 \cos \left(x + \frac{\Delta x}{2}\right) \cdot \sin \left(\frac{\Delta x}{2}\right).$ $\frac{\Delta y}{\Delta x} = 2 \cos \left(x + \frac{\Delta x}{2}\right) \cdot \frac{\sin \frac{\Delta x}{2}}{\Delta x}$ $= 2 \cos \left(x + \frac{\Delta x}{2}\right) \cdot \frac{\sin \left(\frac{\Delta x}{2}\right)}{2x \left(\frac{\Delta x}{2}\right)}$ $\frac{dy}{dx} = \lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \rightarrow 0} 2 \cos \left(x + \frac{\Delta x}{2}\right) \cdot \frac{\sin \frac{\Delta x}{2}}{2x \frac{\Delta x}{2}}$ $= \cos x \cdot 1 = \cos x$	1 1 1 1 1 1		
<u>II</u>	6.	$y = a \cos(\log x) + b \sin(\log x)$ $\frac{dy}{dx} = a \cdot \sin \log x \cdot \frac{1}{x} + b \cdot \cos \log x \cdot \frac{1}{x}$ $x \cdot \frac{dy}{dx} = -a \sin \log x + b \cos \log x$ diff. w.r.t 'x' $x \cdot \frac{d^2y}{dx^2} + \frac{dy}{dx} \cdot 1 = -a \cos \log x \cdot \frac{1}{x} + b \sin \log x \cdot \frac{1}{x}$ $x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} = -[a \cos \log x + b \sin \log x]$ $= -y$	1 1 1 1		

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<u>II.</u>	2.	$\frac{\sin(180+A) \cdot \cos(90-A) \cdot \tan(270+A)}{\sec(540-A) \cdot \cos(360+A)}$ $\sin(180+A) = -\sin A, \cos(90-A) = \sin A,$ $\tan(270-A) = -\cot A, \sec(540-A)$ $= \sec(360+(180-A))$ $= -\sec A$ $\cos(360+A) = \cos A$ $L.H.S = \frac{-\sin A \cdot \sin A \cdot -\cot A}{-\sec A \cdot \cos A} = \frac{-\sin A \cdot \sin A \cdot \frac{\cos A}{\sin A}}{\frac{1}{\cos A} \cdot \cos A}$ $= -\sin A \cos A$	1+1 1+1 (1) (1)		
	3.	$\cos 20 \cdot \cos 40 \cdot \cos 60 \cdot \cos 80 = [\cos 20 \cdot \cos 40] \cdot \frac{1}{2} \cdot \cos 80$ $= \frac{1}{2} \cdot \left[ \frac{1}{2} (\cos 60 + \cos 20) \right] \cdot \cos 80$ $= \frac{1}{4} \cos 60 \cdot \cos 80 + \frac{1}{4} \cos 20 \cdot \cos 80$ $= \frac{1}{8} \cos 80 + \frac{1}{4} \cdot \frac{1}{2} (\cos 100 + \cos 60)$ $= \frac{1}{8} \cos 80 + \frac{1}{8} \cos 100 + \frac{1}{8} \cdot \frac{1}{2}$ $= \frac{1}{8} \cos(180-100) + \frac{1}{8} \cos 100 + \frac{1}{16}$ $= \frac{1}{8} \cos 100 + \frac{1}{8} \cos 100 + \frac{1}{16}$ $= \frac{1}{16}$	(1) (1) (1) (1) (1) (1)		
	4.	$a = 2 \text{ cm}, b = 3 \text{ cm}, c = 4 \text{ cm}$ $A = \cos^{-1} \left( \frac{b^2 + c^2 - a^2}{2bc} \right) = \cos^{-1} \left( \frac{21}{24} \right) = 28^{\circ} 57' 18''$ $B = \cos^{-1} \left( \frac{a^2 + c^2 - b^2}{2ac} \right) = \cos^{-1} \left( \frac{11}{16} \right) = 46^{\circ} 34' 3''$ $C = 180 - (A+B) = 180 - 75^{\circ} 31' 21'' = 104^{\circ} 28' 39''$	2 2 2		

Qn.No	sub.divi sion	Scoring indicator	Split up Score	Sub Total	Total
		$x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} = -y$ $x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + y = 0$	(1)		
<u>II</u>	7.	$y = 3x^2 + x - 2$ $\frac{dy}{dx} = 6x + 1$ $\left(\frac{dy}{dx}\right)_{(1,2)} = 6(1) + 1 = 7$ <p>Eqn of tangent</p> $(y - y_1) = \frac{dy}{dx} (x - x_1)$ $(y - 2) = 7(x - 1)$ $7x - y - 5 = 0$ <p>Eqn of Normal</p> $(y - y_1) = \frac{-1}{\frac{dy}{dx}} (x - x_1)$ $(y - 2) = \frac{-1}{7} (x - 1)$ $x + 7y - 15 = 0$	(1) (1) (1) (1) (1) (1)	30	
<u>III</u>	a.	$\frac{\sin A}{1 - \cos A} + \frac{1 - \cos A}{\sin A} = 2 \csc A$ $\frac{\sin A}{1 - \cos A} + \frac{1 - \cos A}{\sin A} = \frac{\sin^2 A + (1 - \cos A)^2}{\sin A (1 - \cos A)}$ $\frac{\sin^2 A + 1 + 2 \cos^2 A - 2 \cos A}{(\sin A) (1 - \cos A)} = \frac{2 - 2 \cos A}{\sin A (1 - \cos A)} \quad   +1$ $= \frac{2(1 - \cos A)}{\sin A (1 - \cos A)} = \frac{2}{\sin A} = \csc A \quad   +1$	(1)  1+1 1+1		

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III	b.	<p>Consider <math>\triangle ABC</math></p> $\frac{AC}{BC} = \tan 30$ $AC = BC \tan 30$ $= 60 \cdot \frac{1}{\sqrt{3}} = \frac{60}{\sqrt{3}}$ <p><math>AD = 150^\circ</math></p> <p>height of 1<sup>st</sup> tower <math>BE = CD = AD - AC</math></p> $= 150 - \frac{60}{\sqrt{3}}$ $= 115.36$  <p>figure -</p>	<p>(1)</p> <p>(1)</p> <p>(1)+1</p> <p>(1)</p>		
	c.	$3 \sin \theta + 4 \cos \theta = R \sin(\theta + \alpha) \quad \text{--- (1)}$ $3 \sin \theta + 4 \cos \theta = R \sin \theta \cos \alpha + R \cos \theta \sin \alpha \quad \text{--- (1)}$ <p>Equating coefficient of <math>\sin \theta</math></p> $3 = R \cos \alpha \quad \text{--- (2)}$ <p>Equating coefficient of <math>\cos \theta</math></p> $4 = R \sin \alpha \quad \text{--- (3)}$ $\text{(2)}^2 + \text{(3)}^2 \Rightarrow 3^2 + 4^2 = R^2 [\sin^2 \alpha + \cos^2 \alpha]$ $25 = R^2$ $R = \pm 5$ $\frac{\text{(3)}}{\text{(2)}} \Rightarrow \frac{4}{3} = \frac{R \sin \alpha}{R \cos \alpha} = \tan \alpha$ $\Rightarrow \alpha = \tan^{-1} \left( \frac{4}{3} \right)$ <p>Substituting in (1) <math>x = 3 \sin \theta + 4 \cos \theta</math></p> $= \pm 5 \sin(\theta + \tan^{-1} \frac{4}{3}) \quad \text{--- (1)}$	<p>(1)</p> <p>(1)</p> <p>(1)</p> <p>(1)</p> <p>(1)</p>	15	

Qn.No	sub.divi sion	Scoring indicator	Split up Score	Sub Total	Total
<u>IV</u>	a.	$\tan A = \frac{3}{4}, \sin B = \frac{5}{13}$ $\sec^2 A = 1 + \tan^2 A = 1 + \frac{9}{16} = \frac{25}{16}$ $\sec A = \frac{5}{4} \Rightarrow \cos A = \frac{4}{5} \Rightarrow \sin A = \sqrt{1 - \cos^2 A}$ $= \sqrt{1 - \frac{16}{25}} = \frac{3}{5}$ $\sin B = \frac{5}{13} \Rightarrow \cos B = \sqrt{1 - \sin^2 B} = \sqrt{1 - \frac{25}{169}}$ $= \sqrt{\frac{144}{169}} = \frac{12}{13}$ $\sin(A-B) = \sin A \cos B - \cos A \sin B$ $= \frac{3}{5} \times \frac{12}{13} - \frac{4}{5} \times \frac{5}{13}$ $= \frac{36}{65} - \frac{20}{65} = \frac{16}{65}$ $\cos(A+B) = \cos A \cos B - \sin A \sin B$ $= \frac{4}{5} \times \frac{12}{13} - \frac{3}{5} \times \frac{5}{13}$ $= \frac{48}{65} - \frac{15}{65} = \frac{33}{65}$	1+1 1 1	15	
	b.	$\sqrt{\frac{1-\sin A}{1+\sin A}} = \sqrt{\frac{(1-\sin A)(1-\sin A)}{(1+\sin A)(1-\sin A)}}$ $= \frac{(1-\sin A)}{\sqrt{1-\sin^2 A}} = \frac{1-\sin A}{\sqrt{\cos^2 A}} = \frac{1-\sin A}{\cos A}$ $= \frac{1}{\cos A} - \frac{\sin A}{\cos A} = \sec A - \tan A$	1+1+1 1		
	c.	$\theta = 30^\circ, L.H.S = \tan 2\theta = \tan 2 \times 30 = \tan 60 = \sqrt{3}$ $R.H.S = \frac{2 \tan \theta}{1 - \tan^2 \theta} = \frac{2 \tan 30}{1 - \tan^2 30} = \frac{2 \times \frac{1}{\sqrt{3}}}{1 - \frac{1}{3}} = \frac{\frac{2}{\sqrt{3}}}{\frac{2}{3}} = \sqrt{3}$	1+1		

Qn.No	sub.divi sion	Scoring indicator	Split up Score	Sub Total	Total
		$= \frac{2}{\sqrt{3}} \times \frac{3}{2} = \sqrt{3}$ $L.H.S = R.H.S$ <p>Hence verified.</p>	1		
V.	a.	$\frac{\sin 2A + (\sin 5A - \sin A)}{\cos 2A + (\cos 5A + \cos A)} = \frac{\sin 2A + 2\cos 3A \cdot \sin 2A}{\cos 2A + 2\cos 3A \cdot \cos 2A}$ $= \frac{\sin 2A [1 + 2\cos 3A]}{\cos 2A [1 + 2\cos 3A]} = \tan 2A$	1+1 1+1		
	b.	$R(a^2 + b^2 + c^2) = abc(\cot A + \cot B + \cot C)$ $R.H.S = abc \cdot \frac{\cos A}{\sin A} + abc \cdot \frac{\cos B}{\sin B} + abc \cdot \frac{\cos C}{\sin C}$ $= \frac{abc \cdot \cos A}{\sin A} + \frac{abc \cdot \cos B}{\sin B} + \frac{abc \cdot \cos C}{\sin C}$ $= 2R \left[ \frac{b^2 + c^2 - a^2}{2} + \frac{a^2 + c^2 - b^2}{2} + \frac{a^2 + b^2 - c^2}{2} \right]$ $= 2R \left[ \frac{b^2 + c^2 - a^2 + a^2 + c^2 - b^2 + a^2 + b^2 - c^2}{2} \right]$ $= R[a^2 + b^2 + c^2] = L.H.S$	1 1 1 1 1	15	
	c.	$L.H.S = (\sin A + \sin 3A) + (\sin 5A + \sin 7A)$ $= 2\sin 2A \cdot \cos 2A + 2\sin 4A \cdot \cos 4A$ $= 2\cos 2A [\sin 2A + \sin 4A]$ $= 2\cos 2A \cdot 2\sin 4A \cdot \cos 2A$ $= 4\cos 2A \cdot \cos 2A \cdot \sin 4A$	1+1 1 1		

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VI.	a.	$\frac{\sin 3x}{\sin x} - \frac{\cos 3x}{\cos x} = \frac{3\sin x - 4\sin^3 x}{\sin x} - \frac{4\cos^3 x - 3\cos x}{\cos x}$ $= (3 - 4\sin^2 x) - (4\cos^2 x - 3)$ $= 3 - 4\sin^2 x - 4\cos^2 x + 3$ $= 6 - 4[\sin^2 x + \cos^2 x] = 6 - 4 = 2$	1+1 (1) (1) (1)		
	b.	$A = 35^\circ, B = 68^\circ, c = 25 \text{ cm.}$ $C = 180 - (A+B) = 180 - (35^\circ + 68^\circ)$ $= 77^\circ$ By sine rule $\frac{a}{\sin A} = \frac{c}{\sin C}$ $\frac{a}{\sin 35} = \frac{25}{\sin 77} \Rightarrow a = \frac{25 \times \sin 35}{\sin 77}$ $= 14.72 \text{ cm}$ $\frac{b}{\sin B} = \frac{c}{\sin C} \Rightarrow b = \frac{c \sin B}{\sin C} = \frac{25 \times \sin 68}{\sin 77}$ $= 23.79 \text{ cm}$	(1) (1) (1) (1) (1)	15	
	c.	$[\sin 50 - \sin 70] + \cos 80$ $= 2 \cos \frac{50+70}{2} \cdot \sin \frac{50-70}{2} + \cos 80$ $= 2 \cos 60 \cdot \sin^{-10} + \cos 80$ $= 2 \times \frac{1}{2} \cdot \sin 10 + \cos(90-10)$ $= -\sin 10 + \sin 10 = 0$	(1) (1) (1) (1) (1)		



Qn.No	sub.divi sion	Scoring indicator	Split up Score	Sub Total	Total
<u>VIII</u>	a.	$y = (1+x^2)^{10} \sin^2 x$ $\frac{d}{dx}(uv) = u \cdot \frac{dv}{dx} + v \cdot \frac{du}{dx}$ $= (1+x^2)^{10} \frac{d}{dx} \sin^2 x + \sin^2 x \cdot \frac{d}{dx} (1+x^2)^{10}$ $= (1+x^2)^{10} \cdot 2 \sin x \cdot \cos x + \sin^2 x \cdot 10 \cdot (1+x^2)^9 \cdot 2x$ $= \underline{\underline{2(1+x^2)^{10} \sin x \cdot \cos x + 20x(1+x^2)^9 \sin^2 x}}$	(1) (1) 1+1 1		
	b.	$ax^2 + by^2 + 2gx + 2fy + c = 0$ <p>diffn w.r.t. x.</p> $a \cdot 2x + 2y \cdot \frac{dy}{dx} + 2g \cdot 1 + 2f \cdot \frac{dy}{dx} + 0 = 0$ $(2by + 2f) \frac{dy}{dx} = -2ax - 2g$ $\frac{dy}{dx} = \frac{-2(ax+g)}{2(by+f)}$ $= \frac{-ax-g}{by+f}$	(3) (1) (1)	15	
	c.	$x = a(\theta - \sin \theta), y = a(1 - \cos \theta)$ $\frac{dx}{d\theta} = a[1 - \cos \theta], \frac{dy}{d\theta} = a \cdot \sin \theta$ $\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{a \sin \theta}{a(1 - \cos \theta)} = \frac{\sin \theta}{1 - \cos \theta}$	1+1 1+1+1		

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1x.	a.	<p>tangent <math>\parallel^k</math> to x-axis if <math>\frac{dy}{dx} = 0</math></p> $\frac{d}{dx} \left( \frac{x}{x^2+1} \right) = \frac{(x^2+1) \frac{d}{dx} x - x \cdot \frac{d}{dx} (x^2+1)}{(x^2+1)^2}$ $= \frac{(x^2+1) \cdot 1 - x \cdot 2x}{(x^2+1)^2} = \frac{x^2+1-2x^2}{(x^2+1)^2}$ $= \frac{1-x^2}{(x^2+1)^2}$ <p>tangent <math>\parallel^k</math> to x-axis if <math>\frac{dy}{dx} = 0</math></p> $\Rightarrow \frac{1-x^2}{(x^2+1)^2} = 0$ $\Rightarrow 1-x^2 = 0 \Rightarrow x^2 = 1 \Rightarrow x = \pm 1$ <p><math>\therefore</math> tangent <math>\parallel^k</math> to x-axis if <math>x = \pm 1</math></p>	(1)		
	b.	<p><math>S = a \cos nt + b \sin nt</math></p> <p>velocity = <math>v = \frac{ds}{dt} = a(\sin nt)n + b(\cos nt)n</math></p> $= -an \sin nt + bn \cos nt$ <p>acceleration = <math>a = \frac{d^2s}{dt^2} = -an \cos nt \cdot n + bn \sin nt \cdot n</math></p> $= -n^2 a \cos nt - n^2 b \sin nt$ $= -n^2 [a \cos nt + b \sin nt]$ $a = -n^2 S$ <p><math>\therefore</math> acceleration varies as displacement</p>			

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IX.	c.	$y = 2x^3 - 9x^2 + 12x$ $\frac{dy}{dx} = 6x^2 - 18x + 12$ $\frac{d^2y}{dx^2} = 12x - 18$ St. pts are given by $\frac{dy}{dx} = 0 \Rightarrow 6x^2 - 18x + 12 = 0$ $\Rightarrow x^2 - 3x + 2 = 0$ $\Rightarrow x = 2 \& x = 1$ at $x=1$ $\frac{d^2y}{dx^2} = 12 \times 1 - 18 = -6 < 0$ $\therefore$ deflection is max. at $x=1$ Max. deflected op is $y = 2 \times 1^3 - 9 \times 1^2 + 12 \times 1 = 5$	1 1 1 1 1	15	
X	a.	$\frac{dv}{dt} = 20 \text{ cc/sec}$ $v = \frac{4}{3} \pi r^3$ $\frac{dv}{dt} = \frac{4}{3} \times 3\pi r^2 \cdot \frac{dr}{dt} = 4\pi r^2 \cdot \frac{dr}{dt}$ $\frac{dr}{dt} = \frac{20}{4\pi r^2}$ $A = 4\pi r^2$ $\frac{dA}{dt} = 4\pi \cdot 2r \cdot \frac{dr}{dt} = 4\pi \cdot 2r \cdot \frac{20}{4\pi r^2} = \frac{-40}{r} = \frac{-8}{3} \text{ cm}^2/\text{sec}$ $\therefore$ surface area shrinking at the rate of $\frac{8}{3} \text{ cm}^2/\text{sec}$	1 1 1 1 1		

