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(REVISION — 2015)

Reg. No. ....

Signature ....

DIPLOMA EXAMINATION IN ENGINEERING/TECHNOLOGY/ MANAGEMENT/COMMERCIAL PRACTICE — OCTOBER, 2019

THEORY OF STRUCTURES - II

[*Time* : 3 hours

(Maximum marks : 100)

PART — A

(Maximum marks : 10)

Marks

I Answer *all* questions in one or two sentences. Each question carries 2 marks.

1. Define slenderness ratio of a column.
2. Show the limit of eccentricity of a rectangular column section width B and thickness D.
3. Show the elastic curve of a fixed beam due to external loading.
4. Reproduce the equations for slope and deflection for a simply supported beam with central point load.
5. Define distribution factor for a member in moment distribution method. $(5 \times 2 = 10)$

PART — B

(Maximum marks : 30)

II Answer any *five* of the following questions. Each question carries 6 marks.

1. A steel rod 5m long and 40mm diameter is used as a column, with one end fixed and other end free. Determine the crippling load by Euler's formula. Take E as 200GPa.
2. A masonry dam of trapezoidal section is vertical on water face. The width of the dam at the top is 4m and its width at the bottom is 12m. The height of the dam is 23m and it contains water up to a height of 20m above the base. Draw the stress distribution diagram of the dam. Take the specific weight of the dam material is $20kN/m^3$ and specific weight of water is $10kN/m^3$.
3. Explain the following.

   (a) Angle of repose (b) Weep holes (c) Active earth pressure.
4. A simply supported beam of span 6m is subjected to a uniformly distributed load over the entire span. If the deflection at the centre of the beam is not to exceed 4mm, compute the value of the U D load the beam can carry. Take $E = 200 \times 10^3\ N/mm^2$ and $I = 300 \times 10^6\ mm^4$.

2

Marks

5. Compute the maximum slope and deflection of a cantilever beam of length 'L' carrying a point load 'W' at the free end using moment area method.

6. Illustrate the theorem of three moments and reproduce the equation.

7. Explain the following terms in moment distribution method.

   (a) Stiffness factor (b) Carryover moment (c) Unbalanced moment $(5 \times 6 = 30)$

PART — C

(Maximum marks : 60)

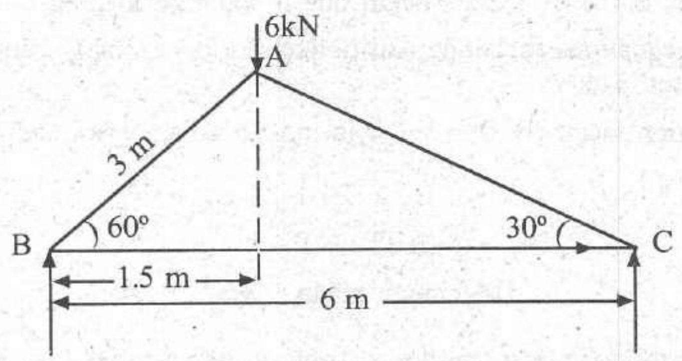
(Answer *one* full question from each unit. Each full question carries 15 marks.)

UNIT — I

III (a) Compute the Euler's crippling load for a hollow cylindrical steel column 38 mm external diameter and 2.5mm thick. The length of the column is 2.3m and hinged at its both ends. Take E = 205GPa. Also determine the crippling load by Rankine's formula using constants.

$\sigma c = 335 N/mm^2$ and $'a' = \frac{1}{7500}$ 8

(b) The truss ABC shown in figure has a span of 6 m. It is carring a load of 6 kN. Find the forces in the members AB, AC and BC.



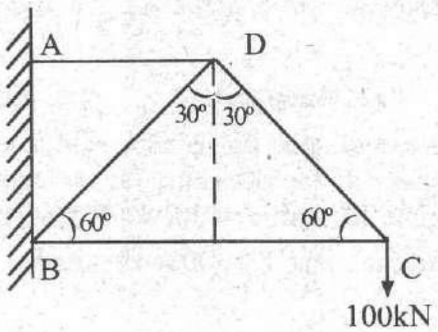
7

OR

IV (a) A solid rectangular column of length 4m is having a cross section of 200mm × 100mm. If the ends of the member is hinged, find the Euler's crippling load.

Take $E = 200kN/mm^2$. 8

(b) Compute the magnitude and nature of forces in the members of the truss shown in figure by method of joints.



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Marks

UNIT — II

V (a) A hollow steel column having 25cm external diameter and 20cm internal diameter. The column carries an eccentric compressive load of 500kN at a distance 10cm from its axis. Determine the maximum tensile and compressive stress. Also draw the stress distribution diagram. 8

(b) A concrete dam of trapezoidal section having vertical water face is 25m height. The width of the dam is 12m at the base and 5m at the top. Compute (i) The resultant pressure on the base per metre length (ii) The point where the resultant pressure cuts the base (iii) Eccentricity of the resultant. The height of the free surface of water above the base is 20m and specific weight of concrete is $25kN/m^3$ and that of water is $10kN/m^3$. 7

OR

VI (a) A rectangular column 20cm wide and 15cm deep is carrying a vertical load of 1000kN at an eccentricity of 5cm in a plane bisecting the depth. Determine the maximum and minimum intensities of stress in section. 8

(b) Compute the maximum and minimum intensities of pressure at the base of a 12m high retaining wall with top width 3m and base width 6m. Specific weight of the soil and wall material are $20kN/m^3$ and $25kN/m^3$ respectively. Assume angle of repose as 30°. 7

UNIT — III

VII (a) A simply supported beam of span 4m is carrying a uniformly distributed load of 2kN/m over the entire span. Find the maximum slope and deflection of the beam. Take $E = 200Gpa$ and $I = 400 \times 10^3\ mm^4$. 8

(b) Explain Macaulay's method of slope and deflection of a simply supported beam with central point load. 7

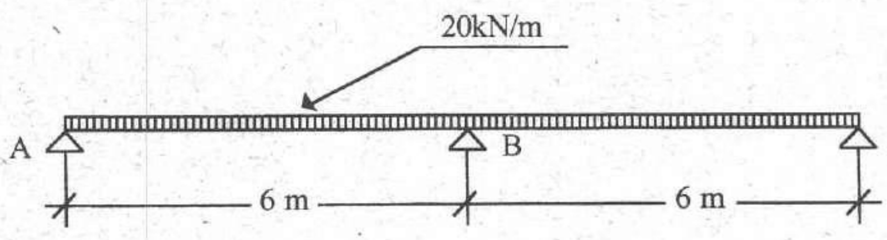
OR

VIII (a) Explain moment area method for finding out slope and deflection of a cantilever beam with uniformly distributed load over the entire span. 8

(b) A beam 3m long simply supported at its ends is carrying a point load at its centre. If the slope at the ends of the beam not to exceed 1°, find the deflection at the centre of the beam. 7

UNIT — IV

IX (a) A continous beam ABC 12m long rests on three supports A, B and C at the same level and is loaded with uniformily distributed load of 20kN/m as shown in figure. Determine the moments over the beam and draw the bending moment diagram. Assume flexural rigidity *EI* as constant.



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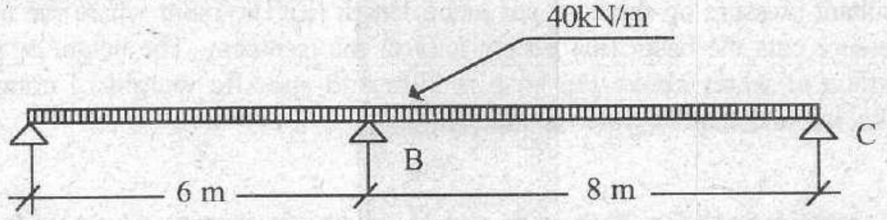
4

Marks

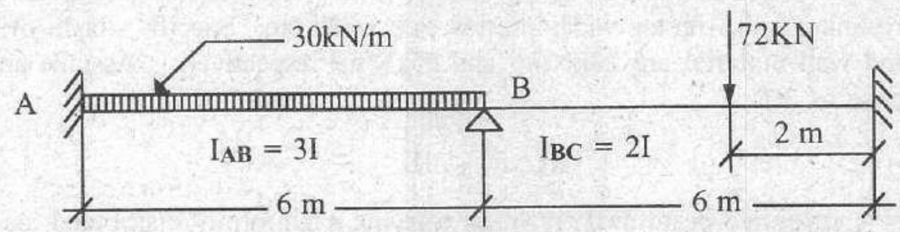
(b) Explain how to find distribution factor for the members OA, OB, OC meet at a rigid point 'O'. The ends 'A & B' are fixed and 'C' is hinged. 7

OR

X (a) A continuous beam ABC 14m long rests on three supports A, B and C as shown in figure. The beam is loaded with uniformly distributed load of 40kN/m. Determine the moments over the beam and draw the bending moment diagram. Assume flexural rigidity *EI* as constant. 8



(b) Analyse the beam shown in figure by moment distribution method and draw the bending moment diagram.



Assume E is constant for the beam. 7

20 D

1

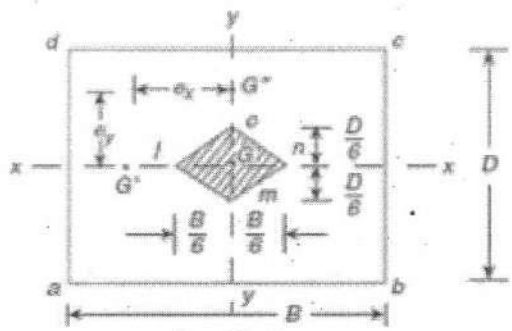
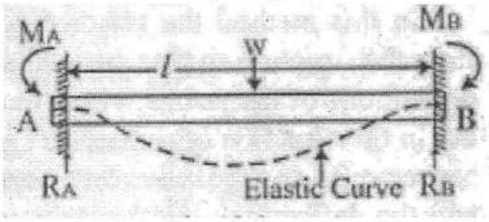
SCORING INDICATORS

Revision : 15

CODE : 4014 - THEORY OF STRUCTURES - II

VERSION: A Revision 2015

Qn.No		Split score	Total score

Qn.No		Split score	Total score
	PART-A		
I. 1	Slenderness ratio (λ) is the ratio of the effective length of a column (Le) and the least radius of gyration (r) about the axis under consideration.	2	
2	Limit of eccentricity of rectangular column sections  	2	
3	 	2	
4	$\theta_B = \frac{-Wl^2}{16EI}$ Radians $\theta_A = \frac{Wl^2}{16EI}$ Radians $Yc = \frac{-Wl^3}{48EI}$	2	
5	**The distribution factor for a member at a joint is the ratio of the stiffness factor of the member to the total stiffness factor of all the members meeting at the ioint.**	2	10
	PART- B		
II) 1	SOLUTION $l = 5m = 5\times10^3 mm$ $d = 40 mm$ $E = 200 GPa = 200\times10^3 N/mm^2$ moment of Inertia, $I = \frac{\pi}{64} d^4$	1	

$= \frac{\pi}{64}(40)^2 = 40000\pi \text{ mm}^4 = 125600 \text{ mm}^4$ 1

Since the column is fixed at one end and other end free 1

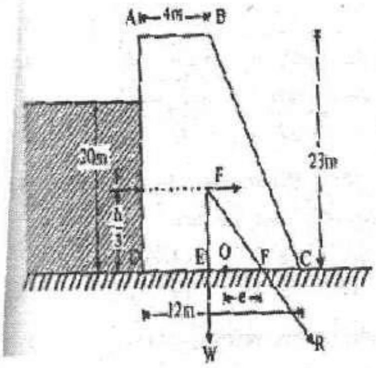
$\therefore L_e = 2l = 2 \times 5 \times 10^3 = 10 \times 10^3$ mm 1

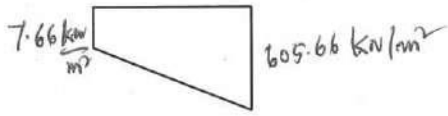
$\therefore$ Eulers Crippling load, $P_E = \frac{\pi^2 E I}{L_e^2}$

$= \frac{\pi^2 (200 \times 10^3) \times 40000\pi}{(10 \times 10^3)^2}$ 6

2 **Solution-Given**

$= \underline{2.48 \text{ kN}}$ 2





Stress diagram 1

H = 23m, h=20m, b=12 m,

a = 4m

w = 10kN/m³

The total pressure of water on the dam per unit length

$$\frac{wh^2}{2} = \frac{10 \times 20^2}{2} = 2000\text{kN}$$ 1

Weight of the dam per unit length

$$W = \frac{(4+12)}{2} 23 \times 1 \times 20 = 3680\text{kN}$$ 1

$$DE = \frac{4 \times 23 \times 2 + \frac{8 \times 23}{2}\left(4 + \frac{8}{3}\right)}{4 \times 23 + \frac{8 \times 23}{2}}$$

= 4.33 m.

The resultant R cuts the base DC at F

$$EF = \frac{F}{W} x \frac{h}{3} = \frac{2000}{3680} x \frac{20}{3} = 3.62 \text{ m}$$

$$DF = DE + EF = 4.33 + 3.62 = 7.95 \text{ m}$$

The above weight W acts through the centre of gravity of the section and cuts the base DC at E.

eccentricity of the resultant, e $= DF - \frac{b}{2}$

$$= 7.95 - \frac{12}{2} = 1.95 \text{ m}$$

1

Maximum stress occurs at the edge C of the base

$$f_{max} = \frac{W}{b}\left[1 + \frac{6e}{b}\right]:$$

$$= \frac{3680}{12}\left[1 + \frac{6 x 1.95}{12}\right] = 605.66 \text{ kN/m}^2 \text{ (Compressive) -}$$

1

Minimum stress occurs at the edge D of the base

$$f_{min} = \frac{W}{b}\left[1 - \frac{6e}{b}\right]$$

$$= \frac{3680}{12}\left[1 - \frac{6 x 1.95}{12}\right]$$

$$= 7.66 \text{ kN/m}^2 \text{ (Compressive) - -}$$

1

3 1.Angle of repose

The minimum angle of the plane at which the body kept on it starts to slide due to its own weight is called angle of repose. The body will begin to move down the plane, if the angle of inclination of the plane is equal to the angle of friction.

2

2.Weep Holes

Provided in earth retaining structures like retaining walls, underpasses, wing walls and other below ground drainage structures.

Weep Hole is provided in these structures to relieve hydrostatic pressure or water pressure on the walls.

Reducing the water pressure on the walls will reduce the structural design demand of the water or earth resisting wall by reducing its thickness as well as reinforcement requirements. 2

3.Active earth pressure

The minimum value of lateral earth pressure exerted by soil on a structure and the wall moves away from the backfill , occurring when the soil is allowed to yield sufficiently to cause its internal shearing resistance along a potential failure surface to be completely mobilized. 2

4

Solution-given

$l = 6m = 6000\ mm$

$I = 300 \times 10^6 mm^4$

$Yc = 4mm$

$$Y_C = \frac{5wl^4}{384EI}$$ 2

$$\frac{5 \times w \times 6000^4}{384 \times 200 \times 10^3 \times 300 \times 10^6}$$ 2

$$4 = 0.281w$$

$$w = 14.2kN/m$$ 2 6

5

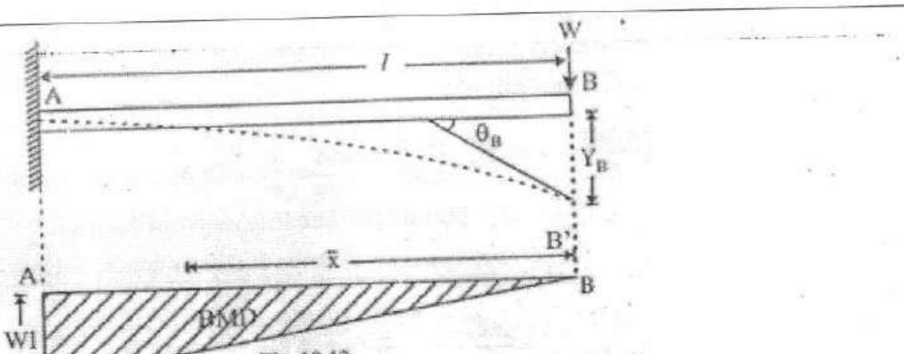


Fig. 18.12

1

Consider a cantilever AB of length l carrying a point load W

Area of bending moment diagram

1

$$A = \frac{wl \times l}{2} = \frac{wl^2}{2}$$

Distance between CG of bending moment diagram and B

1

$$\bar{x} = \frac{2l}{3}$$

According to Mohrs theorem I, change of slope between A and B is

1

$$\theta_B = \frac{A}{EI} = \frac{Wl^2}{2EI} \text{ radians}$$

According to Mohrs theorem II, intercept of the tangents on A and B is

$$BB' = Y_B = \frac{A\bar{x}}{EI} = \frac{Wl^2}{2EI} \times \frac{2l}{3} = \frac{Wl^3}{3EI}$$

2 6

6

The theorem of three moments gives the relationship between the moments at supports in a continuous beam. The theorem states that if AB and BC are any two consecutive spans of a continuous beam subjected to an external loading, the support moment M_a, M_b and M_c are given by the relation. 3

$$M_a l_1 + 2M_b(l_1 + l_2) + M_C l_2 = \frac{6a_1\bar{x}_1}{l_1} + \frac{6a_2\bar{x}_2}{l_2}$$

3 6

7

1.Stiffness Factor- i)It is the moment that must be applied at one end of a constant section member (which is unyielding supports at both ends) to produce a unit rotation of that end when the other end is fixed, i.e. k = 4EI/

ii) It is the moment required to rotate the near end of a prismatic member through a unit angle without translation, the far end being hinged is k = 3EI/l.

2

2. Carry Over Factor
It is the ratio of induced moment to the applied moment

The ratio of moment produced at a joint to the moment applied at the other joint without displacing is called carry over factor. 2

The carry over factor is always (1/2) for members of constant moment of inertia. If the end is hinged/pin connected, the carry over factor is zero.

3. unbalanced moment :- the sum of the fixed end moments meeting at a Joint which is initially clamped is not zero. The algebric sum of the moments meeting at a joint is called unbalanced moment. 2 6

PART-C

III (a) Solution $D = 38$ mm, Thickness $= 2.5$ mm and $d = 38 - 5 = 33$ mm

$l = 2.3 \times 10^3$ mm, $\sigma_c = 335$ N/mm^2, Rankines Constant $a = \frac{1}{7500}$

$I = \frac{\pi}{64}(D^4 - d^4) = \frac{\pi}{64}(38^4 - 33^4) = 14.05 \times 10^3 \pi$ mm^4

Both ends hinged, $L_e = l \times 2.3 \times 10^3$ mm

$\therefore P_E = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2 (205 \times 10^3) \times (14.05 \times 10^3 \pi)}{2.3 \times 10^2} = \underline{16.88 \text{ kN}}$ 4

2. Rankines Crippling Load $P_R = \frac{\sigma_{cs} \cdot A}{1 + a(L_e/k)^2}$

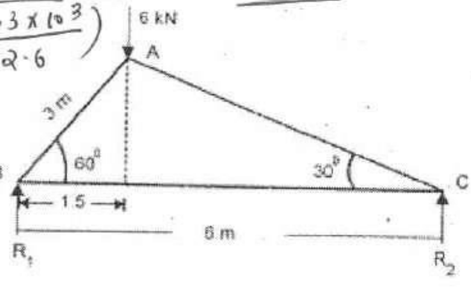
$K = \sqrt{\frac{I}{A}}$

$A = \frac{\pi}{4}(D^2 - d^2) = \frac{\pi}{4}(38^2 - 33^2) = 88.75 \pi$ mm^2

$K = \sqrt{\frac{14.05 \times 10^3 \pi}{88.75 \pi}} = 12.6$ mm

$\therefore P_R = \frac{335 \times 88.75\pi}{1 + \frac{1}{7500}\left(\frac{2.3 \times 10^3}{12.6}\right)^2} = \underline{17.16 \text{ kN}}$ 4 8

(III) (b)

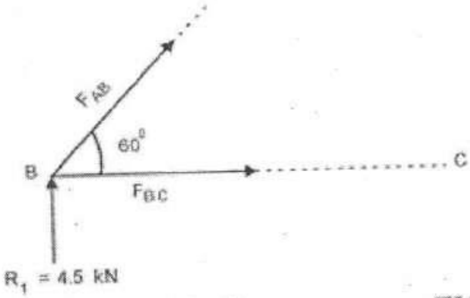


Taking moments about C

$R_1 \times 6 = 6 \times 4.5 \Rightarrow R_1 = 4.5$ kN.

and $R_2 = 6 - 4.5 = 1.5$ kN. 1

Joint B



Resolving vertically, $\Sigma V = 0$

$$4.5 + F_{BA} \sin 60^0 = 0$$

$$F_{BA} = -\frac{4.5}{\sin 60^0} = -5.196 \text{ kN.}$$

1½

$\therefore F_{BA} = F_{AB} = 5.196$ kN (compressive)

Resolving horizontally, $\Sigma H = 0$

$$F_{BA} \cos 60^0 + F_{BC} = 0$$

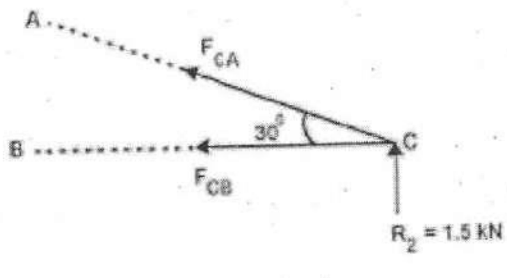
$$-5.196 \cos 60^0 + F_{BC} = 0$$

$$F_{BC} = +2.598 \text{ kN}$$

1½

$\therefore F_{BC} = F_{CB} = 2.598$ kN (tensile).

Joint C



Here, $\Sigma V = 0$

$$F_{CA} \sin 30^0 + 1.5 = 0$$

$$F_{CA} = -3.0 \text{ kN}$$

1½

$F_{CA} = F_{AC} = 3.0$ kN (compressive)

$-F_{CA} \cos 30^0 + F_{CB} = 0$

$-3 \cos 30^0 + F_{CB} = 0$

$F_{CB} = 2.598 \text{ kN} = F_{BC}$; 1½

Results :

Joint	Member	Force (kN)
A	AB	– 5.196
	AC	– 3.0
B	BA	– 5.196
	BC	+ 2.598
C	CA	– 3.0
	CB	2.598

7

IV

(a)

Solution - Given

$l = 12 \text{ m} = 1200 \text{ cm}$

$A = 1 \times 1 = 1 \text{m}^2 = 10^4 \text{cm}^2$

$E = 2 \times 10^4 \text{ KN/cm}^2$

a) Both ends of the column are pinned
Buckling load,

$$P_E = \frac{\pi^2 EI}{L^2} \qquad \text{where } L = l$$ 2

$$P_E = \frac{\pi^2 \times 2 \times 10^4 \times \frac{100 \times 100^3}{12}}{1200^2}$$

$= 1142310.8 \text{kN}$ ---------- 2

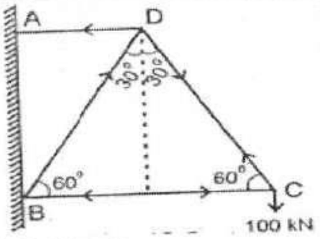
b) One end fixed and other end free

Buckling load, $P_E = \frac{\pi^2 EI}{L^2}$ where $L = 2l$ 2

$$P_E = \frac{\pi^2 \times 2 \times 10^4 \times \frac{100 \times 100^3}{12}}{(2 \times 1200)^2}$$

$= 285577.7 \text{ kN}$ ---------- 2 8

IV. (b)



Solution-

joint C, $\sum H = 0, \sum V = 0$

P_{CD} Sin 60 = 100

$P_{CD} = \frac{100}{\text{Sin } 60} = 115.47\text{kN}$ (tension) 1½

$P_{CB} = P_{CD}$ Cos 60 = 115.47 Cos 60 = 57.73 kN (Compression) 1½

Consider the equlibrium of the joint D, $\sum H = 0, \sum V = 0$

P_{DC} Cos 30 = P_{DB} Cos 30

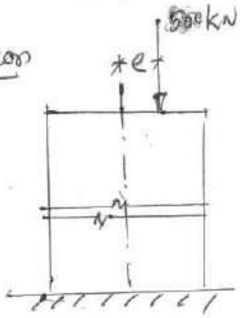
$P_{DC} = P_{DB}$ = 115.47 kN (Compression) 1½

$P_{DA} = P_{DC}$ Sin30 + P_{DB}

Sin 30 = 115.47 Sin 30 + 115.47 Sin 30 = 115.47 kN (Tension) 2½ 7

V (a)

Solution

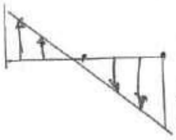


D = 25 cm

d = 20 cm

e = 10 cm

P = 500 kN Fig 1



Direct stress $= \frac{P}{A}$

$= \frac{500 \times 4}{\pi(25^2 - 20^2)} = 2.83 \text{ kN/cm}^2$ 2

Bending stress $= \frac{M \times y}{I} = \frac{Pe \cdot y}{I} = \frac{500 \times 10 \times 12.5 \times 64}{\pi(25^4 - 20^4)}$ 2

$= 5.52 \text{ kN/cm}^2$

$f_{max} = f_d + f_b = 2.83 + 5.52 = 8.35 \text{ kN/cm}^2$ 1

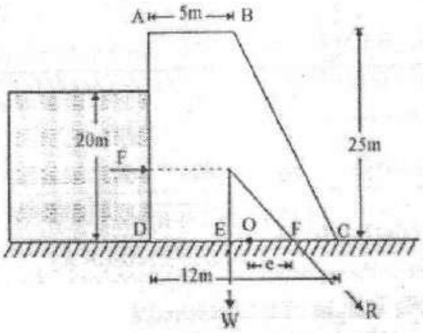
$f_{min} = f_d - f_b = 2.83 - 5.52 = -2.69 \text{ kN/cm}^2$ 1

f_{max} = compression, f_{min} = tensile 1 8

V (b)

Solution-Given

H=25m, , b=12m, a=5m, P=25kN/m³, w=10kN/m³



Weight of the dam per unit length

W = specific weight x volume 1

$$= 25 \times \left(\frac{12+5}{2}\right) 25 \times 1$$

$= 5312.5$ kN

Total pressure of water per unit length of the dam

F = Average intensity of pressure x Area

$$= \left(\frac{wh+o}{2}\right) h \times 1 = \frac{wh^2}{2} = \frac{10 \times 20^2}{2} = 2000\text{kN}$$ 1

i) Resultant pressure on the base per metre length of the cam

$$R = \sqrt{F^2 + W^2} = \sqrt{5312.5^2 + 2000^2} = 5676\text{kN} \text{-------}$$ 1

ii) Take moment about F, $F \times \frac{h}{3} = W \times EF$ 1

$$EF = \frac{F}{W} \times \frac{h}{3} = \frac{2000}{5312.5} \times \frac{20}{3} = 2.5\text{m}$$

$$DE = \frac{5 \times 25 \times 2.5 + \frac{1}{2} \times 7 \times 25\,(5 + \frac{7}{3})}{5 \times 25 + \frac{1}{2} \times 7 \times 25} = 4.49\text{ m}$$ 1

DF = DE + EF = 4.49 + 2.5 = 6.99m 1

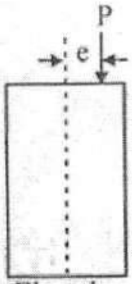
The resultant R cuts the base at a distance 6.99 m from D

iii) eccentricity of the resultant, e $= DF - \frac{b}{2}$

$$= 6.99 - \frac{12}{2} = 0.99\text{ m}$$ 1 7

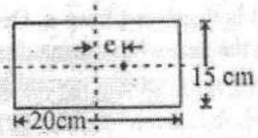
VI

(a)



Elevation

Maximum stress, $f_{max} = \frac{P}{A} + \frac{M x y}{I}$ 1



Plan

$$= \frac{1000}{20 \times 15} + \frac{1000 \times 5 \times 10 \times 12}{15 \times 20^3}$$ 1

$= 8.33\ kN/cm^2$ ------------- 2

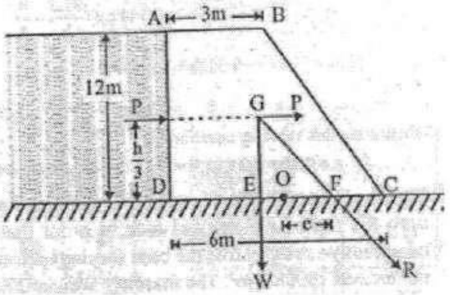
Minimum stress, $f_{min} = \frac{P}{A} - \frac{M x y}{I}$ 1

$$= \frac{1000}{20 \times 15} - \frac{1000 \times 5 \times 10 \times 12}{15 \times 20^3}$$ 2

$= -1.67\ kN/cm^2$ ------------ 1 8

VI

(b)



Solution - Given

$h = 12m$, $a = 3m$, $b = 6m$, $\phi = 30^0$, $w = 20kN/m^3$, $\rho = 25kN/m^3$

Horizontal thrust on the retaining wall per metre length.

$$P = \frac{wh^2}{2} \times \frac{1 - \sin\phi}{1 + \sin\phi} = \frac{20 \times 12^2}{2} \times \frac{1 - \sin 30^0}{1 + \sin 30^0} = 480\ kN$$ 1

Weight of masonry wall per metre length

$$W = \left(\frac{6+3}{2}\right) 12 \times 1 \times 25 = 1350\,\text{kN}$$ 1

Horizontal distance of the CG of the retaining wall from the vertical face AD

$$DE = \frac{a^2+ab+b^2}{3(a+b)} = \frac{3^2+3\times6+6^2}{3(3+6)} = 2.33\,\text{m}$$ 1

$$EF = \frac{P}{w} \times \frac{h}{3} = \frac{480}{1350} \times \frac{12}{3} = 1.42\,\text{m}$$ 1

$$DF = DE + EF = 2.33 + 1.42 = 3.75\,\text{m}$$

eccentricity of the resultant, $e = DF - \frac{b}{2}$

$$= 3.75 - \frac{6}{2} = 0.75\text{m}$$ 1

Maximum stress occurs at the edge C of the base

$$f_{max} = \frac{W}{b}\left[1+\frac{6e}{b}\right] = \frac{1350}{6}\left[1+\frac{6\times0.75}{6}\right]$$

$$= 393.75\,\text{kN/m}^2 \text{ (Compressive)------}$$ 1

Minimum stress occurs at the edge D of the base

$$f_{min} = \frac{W}{b}\left[1-\frac{6e}{b}\right] = \frac{1350}{6}\left[1-\frac{6\times0.75}{6}\right]$$

$$= 56.25\,\text{kN/m}^2 \text{ (Compressive)-------}$$ 1 | 7

VII (a)

Solution

span $l = 4\,m = 4\times10^3\,mm$.

udl, $w = 2\,kN/m$.

$E = 200\,Gpa = 200\times10^3\,N/mm^2$

$I = 400\times10^3\,mm^4$

maximum slope $= \lambda_A = \frac{wl^3}{24EI}$ 2

$$= \frac{2\times(4\times10^3)^3}{24\times200\times10^3\times400\times10^3}$$ 1

Slope $i_A = \underline{0.067 \text{ rad}}$ 1

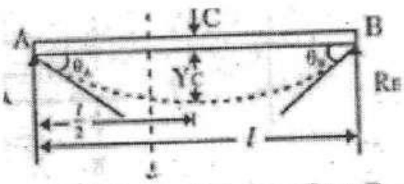
maximum deflection

$$y_c = \frac{5wl^4}{384EI} = \frac{5\times2\times(4\times10^3)^4}{384\times200\times10^3\times400\times10^3}$$ 2+1

$$= \underline{83.33 \text{ mm}}$$ 1 8

VII (b) **Simply supported beam with a central point load**

Consider a simply supported beam AB of length 'l' and carring a point load W at the centre of the beam:



Consider a section XX at a distance x from B 1

BM at section xx, $Mx = \frac{Wx}{2} - W(x - \frac{l}{2})$

According to the differential equation of flexure

$$EI\frac{d^2y}{dx^2} = Mx$$

$$EI\frac{d^2y}{dx^2} = \frac{Wx}{2} - W(x-\tfrac{l}{2})$$

$$EI\frac{dy}{dx} = \frac{Wx^2}{4} + C_1 - \frac{W}{2}(x-\tfrac{l}{2})^2 \text{ -------------(1)}$$ 1

$$EIY = \frac{Wx^3}{12} + C_1x + C_2 - \frac{W}{6}(x-\tfrac{l}{2})^3 \text{ ---------(2)}$$ 1

where C_1 and C_2 are the constant of integration

When $x = 0$, $y = 0$ and consider terms up to dotted line in equation (2), we get $C_2 = 0$, When $x = l$, $y = 0$ and consider all the terms in equation(2)

$$O = \frac{Wl^3}{12} + C_1 l - \frac{W}{6}(l-\tfrac{l}{2})^3$$

$$C_1 l = \frac{-Wl^3}{12} + \frac{W}{6}(\tfrac{l}{2})^3$$

$$= \frac{-Wl^3}{12} + \frac{Wl^3}{48} = \frac{-Wl^3}{16}$$

$$C_1 = \frac{-Wl^2}{16}$$

Substituing this value of C_1 in equation (1)

$$EI\frac{dy}{dx}=\frac{Wx^2}{4}-\frac{Wl^2}{16}-\frac{W}{2}(x-\tfrac{l}{2})^2 \text{ --------(3)}$$ 1

This is the required equation for slope at any section.

When x = 0, $\theta_B=\frac{-Wl^2}{16EI}$ radian

When x = l, $\theta_A=\frac{Wl^2}{16EI}$ radian 1

Substituting the value of C_1 and C_2 in equation (2)

$$EIY=\frac{Wx^3}{12}+\frac{Wl^2x}{16}-\frac{W}{6}(x-\tfrac{l}{2})^3 \text{ -----------(4)}$$ 1

This is the required equation for deflection at any section.

Substituting $x=\frac{l}{2}$ in the equation (4) and consider terms upto dotted line.

$$EIY_C=\frac{W}{12}(\tfrac{l}{2})^3-\frac{Wl^2}{16}(\tfrac{l}{2})=\frac{-Wl^3}{48}$$

$$Y_C=\frac{-Wl^3}{48EI}$$ 1 7

VIII (a) **Cantilever with a uniformly distributed load**

Consider a cantilever AB of length l and carrying a uniformly distributed load of 'w' per unit length.

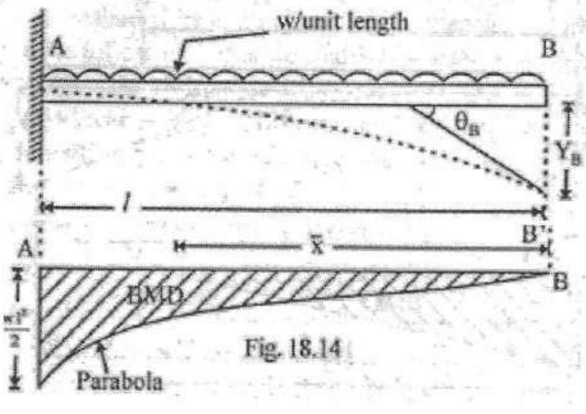


Fig. 18.14 1 1

Area of B.M. diagram = $\frac{wl^2 x\, l}{2}\times\frac{1}{3}=\frac{wl^3}{6}$ 1

Distance between CG of Bending Moment diagram and B 1

$$\bar{x} = \frac{3l}{4}$$

According to Mohrs theorem I, change of slope between A and B' is

$$\theta_B = \frac{A}{EI} = \frac{wl^3}{6EI} \text{ radians}$$ 2

According to Mohrs theorem II, intercept of the tangents on A and B' is

$$BB' = Y_B = \frac{A\bar{x}}{EI} = \frac{wl^4}{8EI}$$ 2 8

VIII (b)

Solution-Given

l = 3m
Slope at ends = 1° = 0.01745 radian

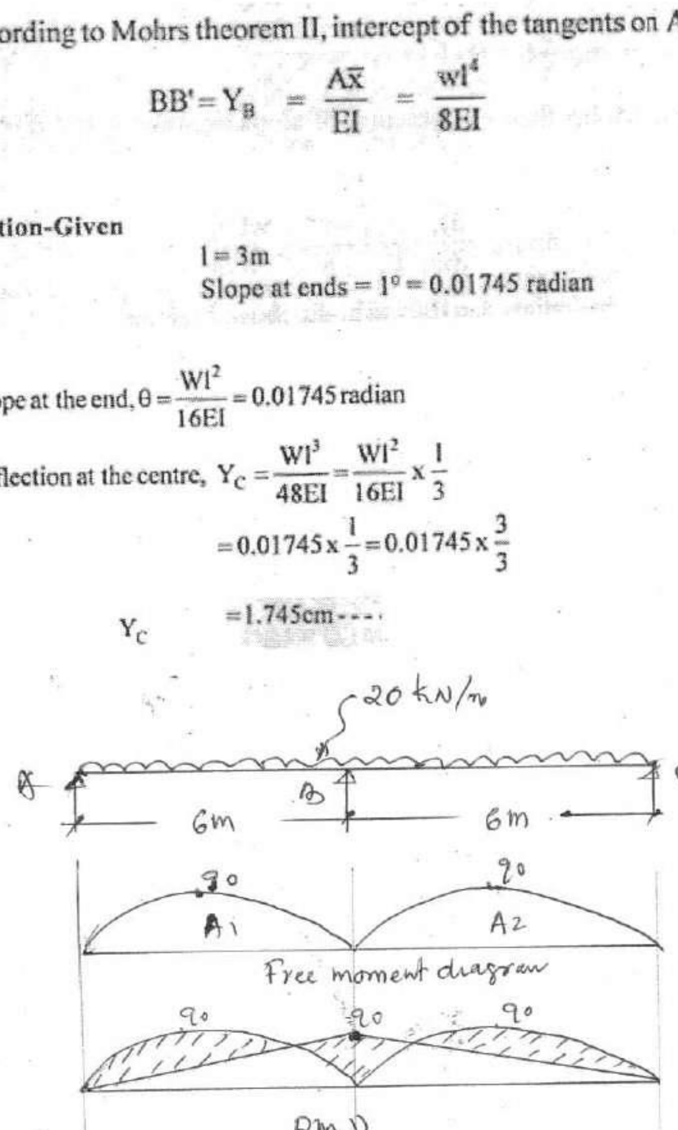
Slope at the end, $\theta = \frac{Wl^2}{16EI} = 0.01745$ radian 2

Deflection at the centre, $Y_C = \frac{Wl^3}{48EI} = \frac{Wl^2}{16EI} \times \frac{l}{3}$ 3

$$= 0.01745 \times \frac{l}{3} = 0.01745 \times \frac{3}{3}$$

Y_C = 1.745cm ---- 2 7

IX (a)



2

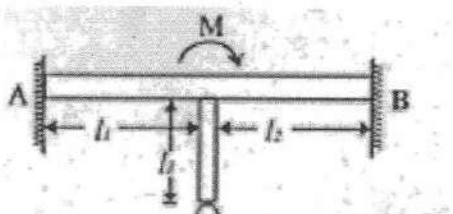
Mid span ordinate of BMD $= \frac{wl^2}{8} = \frac{20 \times 6^2}{8} = 90$ kN m 1

∴ Area of moment diagram
$\bar{x}_1 = \bar{x}_2$ or $A_1 = A_2 = \frac{2}{3} \times 6 \times 90$
$= 360$ units 1

distance of centroid of A_1 from end A and A_2 from end C
$a_1 = a_2 = 3$
EI is same through, $I_1 = I_2$
Clapeyron's theorem
$$M_A L_1 + 2M_B(L_1+L_2) + M_C L_2 = -\frac{6A_1\bar{x}_1}{L_1} - \frac{6A_2\bar{x}_2}{L_2}$$ 1
ie $6M_A + 24M_B + 6M_C = -\frac{6\times 360\times 3}{6} - \frac{6\times 360\times 3}{6}$ 1
since support A & C are simple supports
$M_A = M_C = 0$
$\therefore 24M_B = \frac{6\times 360\times 3}{6} - \frac{6\times 360\times 3}{6}$
$M_B = -90$ kN-m
$= 90$ kN m (hogging) 2 8

IX (b)

Consider three members OA, OB and OC meet at a rigid joint O. Let the ends A and B be fixed and let the end C be hinged.



1

Let M be the moment applied at joint O. Since O is rigid joint, all the members rotate by same angle, say θ. Let M_1, M_2 and M_3 moments shared by members OA, OB and OC.

$M_1+M_2+M_3=M$ 1

Let k1,k2 and k4 be the stiffness and l_1, l_2 and l_3 be the length of members

$\theta = M_1/k_1 = M_2/k_2 = M_3/k_3$ 1

$= \frac{M}{\sum k}$ 1

$\therefore M_i = ki \frac{M}{\sum ki}$

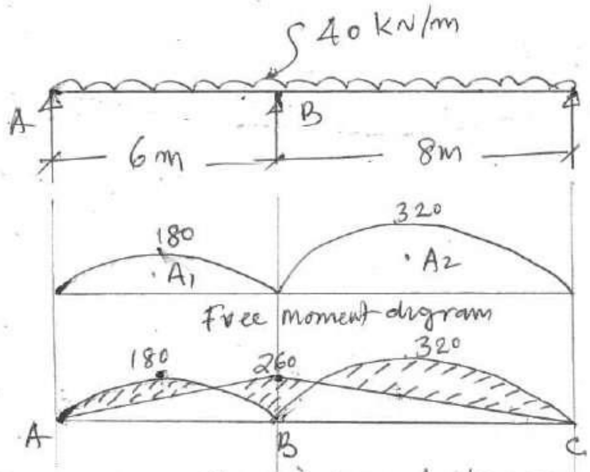
$= \frac{ki}{\sum ki} M$ 1

Distribution factor for OA, $DF_{OA} = \frac{k_{OA}}{\sum k_O}$

Distribution factor for OB, $DF_{OB} = \frac{k_{OB}}{\sum k_O}$

Distribution factor for OC, $DF_{OC} = \frac{k_{OC}}{\sum k_O}$ 2 7

X (a)



Bending moment diagram 2

Mid span moment in AB

$= \frac{wl^2}{8} = \frac{40 \times 6^2}{8} = 180 \text{ kNm}$ 1/2

midspan moment in BC $= \frac{wl^2}{8} = \frac{40 \times 8^2}{8} = 320 \text{ kNm}$ 1/2

$\therefore A_1$ = Area of free BM diagram in span AB

$A_1 = \frac{2}{3} \times 6 \times 180 = 720$ units 1/2

$\bar{x}_1 = 3m$. 1/2

A_2 = Area of free BM diagram in span BC

$A_2 = \frac{2}{3} \times 8 \times 320 = 1706.67$ units 1/2

$\bar{x}_2 = 4 m$. 1/2

Flexural rigidity EI is Constant

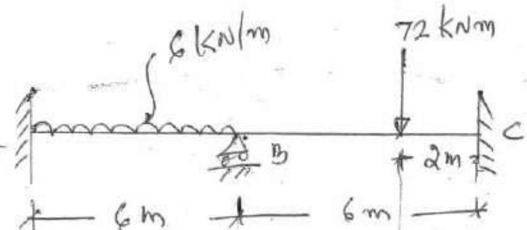
$M_A \times 6 + 2M_B(6+8) + M_C \times 8 = -\frac{6 \times 720 \times 3}{6} - \frac{6 \times 1706.67 \times 4}{8}$ 2

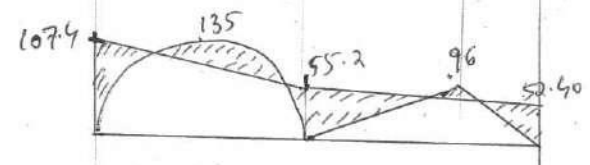
Since ends A and C are simply supported, $M_A = M_C = 0$

$\therefore 2 \times 14 \, M_B = -\dfrac{6 \times 720 \times 3}{6} - \dfrac{6 \times 1706.67 \times 4}{8}$ 2

$\therefore M_B = -260$ kNm (hogging) 1 8

X (b)





1

Fixed end moments

$M_{FAB} = -\dfrac{30 \times 6^2}{12} = -90$ kN-m ½

$M_{FBA} = 90$ kNm

$M_{FBC} = \dfrac{-72 \times 4 \times 2^2}{6^2} = -32$ kNm ½

$M_{FCB} = \dfrac{72 \times 4^2 \times 2}{6^2} = 64$ kNm ½

Distribution factors

$K_{BA} = \dfrac{4E(3I)}{6} = 2EI = \dfrac{2}{3.33} = 0.6$ ½

$K_{BC} = \dfrac{4E(2I)}{6} = 1.33EI = \dfrac{1.33}{3.33} = 0.40$ ½

$\Sigma K = 3.33 EI$

A	B		C
	0.6	0.40	
-90	90	-32	64
	-34.80	-23.20	
-17.40			-11.60
-107.40	55.20	-55.20	52.40

2½

Qst.no	Scoring indicator	Split up score	Sub total	total
	For span AB, free BM diagram = $\frac{wl^2}{8}$			
	$= \frac{30 \times 6^2}{8} = 135$ kNm	½		
	For span BC; free BM diagram = $\frac{Wab}{l}$			
	$= \frac{72 \times 4 \times 2}{6} = 96$ kNm	½		7