## Scoring Indicators

**COURSE NAME: MATHEMATICS II**

**COURSE CODE: 2002** **QID: 2106220096 A**

| Q No | Scoring Indicators | Split score | Sub total | Total score |
|---|---|---|---|---|
| | PART A | | | 9 |
| I.1. | $\sin^2 x + \cos^2 x = 1$ | 1 | 1 | |
| I.2 | $A+B = \begin{bmatrix} 0+3 & 0+7 \\ 2+4 & 1+8 \end{bmatrix} = \begin{bmatrix} 3 & 7 \\ 6 & 9 \end{bmatrix}$ | 1 | 1 | |
| I.3. | $3A = \begin{bmatrix} 3\times1 & 3\times4 & 3\times3 \\ 3\times2 & 3\times1 & 3\times6 \\ 3\times-1 & 3\times2 & 3\times0 \end{bmatrix} = \begin{bmatrix} 3 & 12 & 9 \\ 6 & 3 & 18 \\ -3 & 6 & 0 \end{bmatrix}$ | 1 | 1 | |
| I.4 | $\vec{PQ} = (3-1)\hat{i} + (5-2)\hat{j} + (6-3)\hat{k}$ $= 2\hat{i} + 3\hat{j} + 3\hat{k}$ | 1 | 1 | |
| I.5 | $\lvert\vec{a}\rvert = \sqrt{x^2+y^2+z^2} = \sqrt{2^2+(-3)^2+1^2} = \sqrt{14}$ | 1 | 1 | |
| I.6 | $\int \sec x \tan x \, dx = \sec x + c$ | 1 | 1 | |
| I.7 | $\int e^{2x} dx = \frac{e^{2x}}{2} + c$ | 1 | 1 | |
| I.8 | Order = 3, degree = 1 | 1 | 1 | |
| I.9 | $dy = x\,dx$ $\therefore y = \frac{x^2}{2} + c$ | 1 | 1 | |

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| II | PART B | | | 8 x 3 |
| II.1. | $x^2 - 36 = 16 - 3$<br>$x^2 = 49$<br>$x = \pm 7$ | 1<br>1<br>1 | 3 | |
| II.2. | $A^T = \begin{bmatrix} 1 & 2 & -1 \\ 4 & 1 & 2 \\ 3 & 6 & 0 \end{bmatrix}$<br>$A + A^T = \begin{bmatrix} 2 & 6 & 2 \\ 6 & 2 & 8 \\ 2 & 8 & 0 \end{bmatrix}$<br>$A - A^T = \begin{bmatrix} 0 & 2 & 4 \\ -2 & 0 & 4 \\ -4 & -4 & 0 \end{bmatrix}$ | 1<br>1<br>1 | 3 | |
| II.3. | $\lvert A \rvert = \begin{vmatrix} 1 & 2 \\ 4 & 9 \end{vmatrix} = 9 - 8 = 1$<br>Adjoint of $A = \begin{bmatrix} 9 & -2 \\ -4 & 1 \end{bmatrix}$<br>$A^{-1} = \frac{1}{\lvert A \rvert}$ Adjoint of $A = \begin{bmatrix} 9 & -2 \\ -4 & 1 \end{bmatrix}$ | 1<br>1<br>1 | 3 | |

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| II.4 | $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 2 \\ 6 & -3 & 2 \end{vmatrix}$ | 1 | | |
| | $= (4-3)\hat{i} - (4+6)\hat{j} + (-6-12)\hat{k}$ | 1 | | |
| | $= \hat{i} - 10\hat{j} - 18\hat{k}$ | 1 | 3 | |
| II.5. | $x = 2,\ y = 4,\ z = -3$ | 1+1+1 | 3 | |
| II.6 | $x^2 + 1 = u,\ du = 2x\,dx$ | 1 | | |
| | $\therefore \int \frac{2x}{x^2+1}\,dx = \int \frac{du}{u}$ | | | |
| | $= \log u + c = \log(x^2+1) + c$ | 1+1 | 3 | |
| II.7. | $\int uv\,dx = u\int v\,dx - \int\left[\frac{d}{dx}u\left(\int v\,dx\right)\right]dx$ | 1 | | |
| | $\therefore \int x e^x\,dx = x\int e^x\,dx - \int \frac{d}{dx}(x)\left(\int e^x dx\right)dx$ | | | |
| | $= x e^x - \int 1 \cdot e^x\,dx$ | 1 | | |
| | $= x e^x - e^x + c$ | 1 | 3 | |
| II.8 | $\int \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1}x + c$ | 1 | | |
| | $\therefore \int_0^1 \frac{1}{\sqrt{1-x^2}}\,dx = (\sin^{-1}x)_0^1 = \sin^{-1}(1) - \sin^{-1}(0)$ | 1 | | |
| | $= \frac{\pi}{2}$ | 1 | 3 | |

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| II.9 | Area $= \int_a^b y\,dx$ | 1 | | |
| | $= \int_1^3 (2x+3)\,dx$ | 1 | | |
| | $= \left(2\frac{x^2}{2} + 3x\right)_1^3$ | 1 | | |
| | $= 9+9-(1+3) = 14$ sq.units | 1 | 3 | |
| II.10 | $\frac{dx}{1+x^2} = \frac{dy}{1+y^2}$ | 1 | | |
| | $\therefore \int \frac{dx}{1+x^2} = \int \frac{dy}{1+y^2}$ | 1 | | |
| | $\therefore \tan^{-1}x = \tan^{-1}y + c$ | 1 | 3 | |
| III | PART C | | | 6×7 |
| III.1 | $\Delta = \begin{vmatrix} 1 & 2 & -1 \\ 3 & 1 & 1 \\ 1 & -1 & 2 \end{vmatrix} = -3$ | 2 | | |
| | $\Delta_1 = -3, \quad \Delta_2 = 3, \quad \Delta_3 = -6$ | 3 | | |
| | $x = \frac{\Delta_1}{\Delta}, \quad y = \frac{\Delta_2}{\Delta}, \quad z = \frac{\Delta_3}{\Delta}$ | 1 | | |
| | $x = 1, \quad y = -1, \quad z = 2$ | 1 | 7 | |

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| | OR | | | |
| III.2 | $A = \begin{bmatrix} 5 & 2 \\ 2 & -1 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \end{bmatrix}$, $B = \begin{bmatrix} 4 \\ 7 \end{bmatrix}$ | 1 | | |
| | $AX = B$ | 1 | | |
| | $\lvert A \rvert = -5 - 4 = -9$ | 1 | | |
| | Adjoint matrix $= \begin{bmatrix} -1 & -2 \\ -2 & 5 \end{bmatrix}$ | | | |
| | $A^{-1} = \frac{1}{-9}\begin{bmatrix} -1 & -2 \\ -2 & 5 \end{bmatrix}$ | 1 | | |
| | $\therefore X = A^{-1}B$ | 1 | | |
| | $= \frac{1}{-9}\begin{bmatrix} -1 & -2 \\ -2 & 5 \end{bmatrix}\begin{bmatrix} 4 \\ 7 \end{bmatrix}$ | | | |
| | $= \frac{1}{-9}\begin{bmatrix} -4-14 \\ -8+35 \end{bmatrix}$ | 1 | | |
| | $= \frac{1}{-9}\begin{bmatrix} -18 \\ 27 \end{bmatrix} = \begin{bmatrix} \frac{-18}{-9} \\ \frac{27}{-9} \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \end{bmatrix}$ | 1 | | |
| | | | 7 | 7 |
| | $\therefore x = 2, y = -3$ | | | |

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| III.3 | $\vec{F} = 2\hat{i} - 5\hat{j} + 6\hat{k} + -\hat{i} + 2\hat{j} - \hat{k} + 2\hat{i} + 7\hat{j}$ | 1 | | |
| | $= 3\hat{i} + 4\hat{j} + 5\hat{k}$ | 1 | | |
| | $\vec{AB} = 6\hat{i} + \hat{j} - 3\hat{k} - (4\hat{i} - 3\hat{j} - 2\hat{k})$ | 1 | | |
| | $= 2\hat{i} + 4\hat{j} - \hat{k}$ | 1 | | |
| | Total Workdone $= \vec{F} \cdot \vec{AB}$ | 1 | | |
| | $= (3\hat{i} + 4\hat{j} + 5\hat{k}) \cdot (2\hat{i} + 4\hat{j} - \hat{k})$ | | | |
| | $= 6 + 16 - 5$ | 1 | | |
| | $= 17$ units | 1 | 7 | |
| | OR | | | |
| III.4 | $3\vec{a} = 3(2\hat{i} + 3\hat{j} + 4\hat{k}) = 6\hat{i} + 9\hat{j} + 12\hat{k}$ | 1 | | |
| | $4\vec{b} = 4(-\hat{i} + 3\hat{j} + 2\hat{k}) = -4\hat{i} + 12\hat{j} + 8\hat{k}$ | 1 | | |
| | $3\vec{a} + 4\vec{b} = 2\hat{i} + 21\hat{j} + 20\hat{k}$ | 1 | | |
| | $\lvert 3\vec{a} + 4\vec{b} \rvert = \sqrt{2^2 + (21)^2 + 20^2} = \sqrt{845}$ | 2 | | |
| | unit vector in the direction of $3\vec{a} + 4\vec{b}$ is $\frac{3\vec{a} + 4\vec{b}}{\lvert 3\vec{a} + 4\vec{b} \rvert}$ | 1 | | |
| | $= \frac{1}{\sqrt{845}}(2\hat{i} + 21\hat{j} + 20\hat{k})$ | 1 | 7 | 7 |

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| III.5 (i) | $\vec{a}\cdot\vec{b} = (2\hat{i}+3\hat{j}-\hat{k})\cdot(3\hat{i}-\hat{j}+\hat{k})$ | | | |
| | $= 6-3-1 = 2$ | 1+1 | 2 | |
| III 5 (ii) | $\vec{F} = \hat{i}+2\hat{j}+\hat{k}$ | | | |
| | $\vec{r} = \vec{PA} = (2\hat{i}+3\hat{j}+\hat{k})-(\hat{i}+2\hat{j}-\hat{k})$ | | | |
| | $= \hat{i}+\hat{j}+2\hat{k}$ | 1 | | |
| | $\vec{r}\times\vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 2 \\ 1 & 2 & 1 \end{vmatrix}$ | 1 | | |
| | $= (1-4)\hat{i}-(1-2)\hat{j}+(2-1)\hat{k}$ | | | |
| | $= -3\hat{i}+\hat{j}+\hat{k}$ | 1 | | |
| | Moment $= \lvert\vec{r}\times\vec{F}\rvert$ | 1 | | |
| | $= \sqrt{9+1+1} = \sqrt{11}$ | 1 | 5 | 7 |
| | OR | | | |
| III.6 (i) | Angle between $\vec{a}$ & $\vec{b}$ | | | |
| | $= \cos^{-1}\left(\frac{\vec{a}\cdot\vec{b}}{\lvert\vec{a}\rvert\lvert\vec{b}\rvert}\right)$ | 1 | | |
| | $= \cos^{-1}\left(\frac{10}{5\times4}\right) = \cos^{-1}\left(\frac{1}{2}\right) = 60^\circ$ | 1 | 2 | |

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| III 6. (ii) | $\vec{a} = \hat{i}+\hat{j}+\hat{k}$ $\quad \vec{b} = \hat{i}+3\hat{j}-\hat{k}$ | | | |
| | $\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{vmatrix}$ | 1 | | |
| | $= (-1-3)\hat{i} - (-1-1)\hat{j} + (3-1)\hat{k}$ | | | |
| | $= -4\hat{i}+2\hat{j}+2\hat{k}$ | 1 | | |
| | $\lvert\vec{a}\times\vec{b}\rvert = \sqrt{(-4)^2+2^2+2^2} = \sqrt{24}$ | 1 | | |
| | Unit vector perpendicular to $\vec{a}$ & $\vec{b}$ is | | | |
| | $\dfrac{\vec{a}\times\vec{b}}{\lvert\vec{a}\times\vec{b}\rvert} = \dfrac{-4\hat{i}+2\hat{j}+2\hat{k}}{\sqrt{24}}$ | 1+1 | 5 | 7 |
| III 7. (i) | $\int(3x^2-4x+6)\,dx = 3\int x^2dx - 4\int x\,dx + 6\int dx$ | 1 | | |
| | $= 3\cdot\dfrac{x^3}{3} - 4\cdot\dfrac{x^2}{2} + 6x + c$ | 1 | | |
| | $= x^3 - 2x^2 + 6x + c$ | 1 | 3 | |
| III 7. (ii) | $\int u\cdot v\,dx = u\int v\,dx - \int \dfrac{d(u)}{dx}\left(\int v\,dx\right)dx$ | 1 | | |
| | $\int x\log x\,dx = \int \log x\cdot x\,dx$ | 1 | | |
| | $= \log x\cdot\dfrac{x^2}{2} - \int \dfrac{1}{x}\dfrac{x^2}{2}\,dx$ | 1 | | |
| | $= \log x\cdot\dfrac{x^2}{2} - \dfrac{x^2}{4} + c$ | 1 | 4 | 7 |

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| | **[OR]** | | | |
| III.8 (i) | $\int \frac{1}{1+x^2}\,dx = \tan^{-1}x + c$ | 1 | | |
| | $\therefore \int_0^1 \frac{1}{1+x^2}\,dx = \left(\tan^{-1}x\right)_0^1$ | | | |
| | $= \tan^{-1}1 - \tan^{-1}0$ | 1 | | |
| | $= \frac{\pi}{4}$ | 1 | 3 | |
| III.8 (ii) | $x^2 = u, \quad du = 2x\,dx$ | 1 | | |
| | $\therefore \int x \sec(x^2)\tan x^2\,dx = \int \sec u \tan u \frac{du}{2}$ | 1 | | |
| | $= \frac{1}{2}\sec u + c$ | 1 | | |
| | $= \frac{1}{2}\sec(x^2) + c$ | 1 | 4 | 7 |
| III.9 | $1+\sin x = u, \quad \cos x\,dx = du$ | 1 | | |
| | $\therefore \int \frac{\cos x}{1+\sin x}\,dx = \int \frac{du}{u}$ | | | |
| | $= \log u + c$ | 1 | | |
| | $= \log(1+\sin x) + c$ | 1 | | |
| | $\therefore \int_0^{\pi/2} \frac{\cos x}{1+\sin x}\,dx = \left[\log(1+\sin x)\right]_0^{\pi/2}$ | 1 | | |
| | $= \log\left(1+\sin\frac{\pi}{2}\right) - \log(1+\sin 0)$ | 1 | | |
| | $= \log 2 - \log 1 = \log 2$ | 2 | 7 | |

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| | **OR** | | | |
| III. 10 | (i) $\tan x = u$, $du = \sec^2 x\, dx$ | 1 | | |
| | $\therefore \int e^{\tan x} \sec^2 x\, dx \to \int e^u du$ | | | |
| | $= e^u + c$ | 1 | | |
| | $= e^{\tan x} + c$ | 1 | 3 | |
| III. 10 | (ii) $\cos^2 x = \frac{1+\cos 2x}{2}$ | 1 | | |
| | $\therefore \int \cos^2 x\, dx = \int \frac{1+\cos 2x}{2} dx$ | | | |
| | $= \frac{1}{2}\left[x + \frac{\sin 2x}{2}\right] + c$ | 1 | | |
| | $\therefore \int_0^{\pi} \cos^2 x\, dx = \frac{1}{2}\left[x + \frac{\sin 2x}{2}\right]_0^{\pi}$ | | | |
| | $= \frac{1}{2}\left[\pi + \frac{\sin 2\pi}{2} - \left(0 + \frac{\sin 0}{2}\right)\right]$ | 1 | | |
| | $= \frac{1}{2}\pi$ | 1 | 4 | 7 |
| III. 11 | $x^2 + x = 0$ | 1 | | |
| | ie, $x(x+1) = 0 \Rightarrow x = 0$ or $x = -1$ | 1 | | |
| | $\therefore$ Area $= \int_a^b f(x)\, dx = \int_{-1}^{0} (x^2 + x) dx$ | 1+1 | | |
| | $= \left(\frac{x^3}{3} + \frac{x^2}{2}\right)_{-1}^{0} = 0 + 0 - \left(\frac{-1}{3} + \frac{1}{2}\right)$ | 1+1 | | |
| | $= \frac{1}{6}$ sq. units | 1 | 7 | |

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| | **OR** | | | |
| III.12 | $P = \cot x$, $Q = 2\cos x$ | 1 | | |
| | $\int P\,dx = \int \cot x\,dx = \log(\sin x) + c$ | 1 | | |
| | $\therefore$ Integrating factor $(I.F) = e^{\int P\,dx}$ | 1 | | |
| | ie, $IF = e^{\log(\sin x)}$ | | | |
| | $= \sin x$ | 1 | | |
| | $\therefore$ The solution is | | | |
| | $y.(IF) = \int Q.(IF)\,dx$ | 1 | | |
| | ie, $y.\sin x = \int 2\cos x.\sin x\,dx$ | | | |
| | $= \int \sin 2x\,dx$ | 1 | | |
| | ie, $y.\sin x = -\frac{\cos 2x}{2} + c$ | 1 | 7 | 7 |