TED (15) – 3013 Reg. No. ....

(REVISION — 2015) Signature ....

DIPLOMA EXAMINATION IN ENGINEERING/TECHNOLOGY/ MANAGEMENT/COMMERCIAL PRACTICE — OCTOBER, 2017

THEORY OF STRUCTURES - I

[*Time* : 3 hours

(Maximum marks : 100)

PART — A

(Maximum marks : 10)

Marks

I Answer *all* questions in one or two sentences. Each question carries 2 marks.

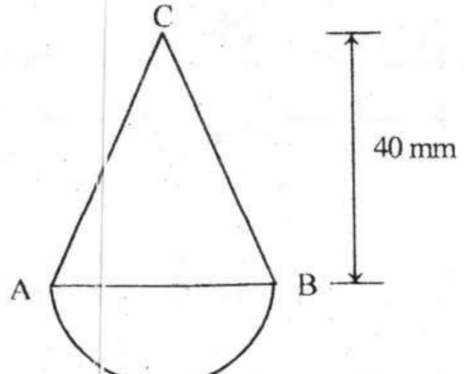
1. List the characteristics of a force.
2. Define radius of gyration.
3. State Poisson's ratio.
4. Define the term torque.
5. State moment of resistance. $(5 \times 2 = 10)$

PART — B

(Maximum marks : 30)

II Answer any *five* of the following questions. Each question carries 6 marks.

1. Calculate the support reactions of a simply supported beam of 4m span with a Point Load of 10kN at its Centre of span and a u.d.l of 2kN/m thought its span.
2. Determine the Centre of Gravity of the solid body consists of right circular cone placed on a solid hemisphere as shown in figure from C.



Marks

3. An alloy bar lm long and 200mm^2 in cross section area is subjected to a compressive force of 20kN. If the modulus of elasticity for the alloy is 100GPa, find the decrease in length of the bar.

4. Define the terms :

   (i) Volumetric strain (ii) Bulk modulus (iii) Modulus of rigidity.

5. Determine the maximum shear stress developed, if the average torque transmitted by a shaft is 2255Nm. The maximum torque is 40% more than the average torque and the diameter of the shaft is 80mm.

6. Differentiate longitudinal stress and hoop stress in thin cylinders.

7. List the assumptions in the theory of simple bending. (5 × 6 = 30)

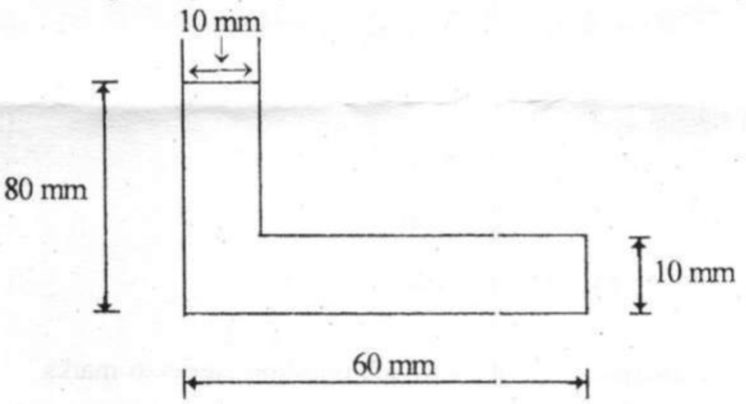
PART — C

(Maximum marks : 60)

(Answer *one* full question from each Unit. Each full question carries 15 marks.)

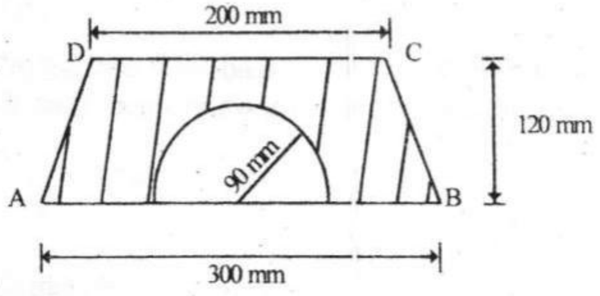
UNIT — I

III (a) Calculate the moment of inertia of the 'L' section shown in figure about a vertical axis passing through its centre of gravity.



8

(b) Determine the centroid of the lamina shown in figure from AB



7

OR

IV (a) Calculate the support reactions of a simply supported beam AB of span 4m with a uniformly varying load of 2kN/m at right support B to 8kN/m at the left support A. 8

(b) Determine the polar moment of inertia of a rectangular section of 200mm width and 300mm depth. 7

Marks

Unit — II

V (a) Define the terms :

(i) Elasticity (ii) Hardness (iii) Ductility (iv) Stiffness. 8

(b) A brass rod 2m long is fixed at its two ends. If the thermal stress is not to exceed 76.5MPa, calculate the temperature through which the rod can be heated. Take $\alpha = 17 \times 10^{-6}/^{\circ}C$ and E = 90GPa. 7

Or

VI (a) A metal bar 50mm × 50mm section is subjected to an axial compressive load of 500kN. The contraction for a 200mm gauge length is found to be 0.5mm and increase in thickness is 0.04mm. Find the values of young's modulus and Poisson's ratio. 8

(b) A metallic bar of 500mm × 200mm and 2m long is subjected to a load of 150kN applied gradually on it. If the stress at elastic limit of the bar material is $200N/mm^2$, determine

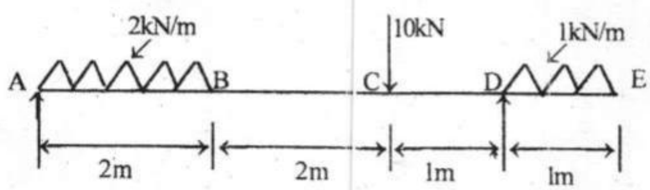
(i) Strain energy

(ii) Proof resilience

(iii) Modulus of resilience E = $200\ kN/mm^2$. 7

Unit — III

VII (a) Sketch SFD and BMD for an overhanging beam shown in figure and calculate the maximum bending moment.



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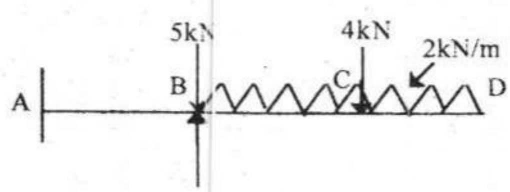
(b) Calculate the maximum torque that can be safely applied to a shaft of 80mm diameter. The permissible angle of twist is 1.5 degree for a length of 5m and shear stress not to exceed 42MPa. Take N = 84GPa. 7

Or

VIII (a) Sketch SFD and BMD of a cantilever beam shown in figure.



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(b) A spherical shell of 2m diameter is made up of 10mm thick plates. Calculate the change in diameter and volume of the shell, when it is subjected to an internal pressure of 1.6MPa. Take E = 200GPa and 1/m = 0.3. 7

Marks

UNIT — IV

IX (a) A beam of 200mm × 400mm in cross section is simply supported at the two ends. It carries a u.d.l of 10kN/m over the entire span. Find the maximum permitted span, if the maximum bending stress permitted is 50N/mm². 8

(b) Derive a formula for shear stress at the section of a loaded beam. 7

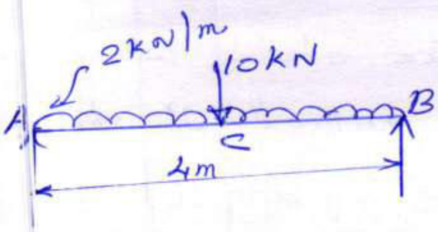
OR

X (a) Derive the equation for simple bending. 8

(b) Calculate the maximum shear stress at the section of a simply supported beam of rectangular Section of size 200mm × 300mm, if the shear force at the section is 100kN. Also calculate the Shear stress at a point 50mm above Neutral axis. 7

2017 ✓ Scoring Indicators (15) 3013

Code : 3013 - Theory of Structures - I Version: B

Qn. No.	Scoring Indicators	Split score	Total score
	Part A		
I. 1.	Magnitude, direction, nature of force, the point at which the force acts on the body.	$4 \times \frac{1}{2} = 2$	
3.	The ratio of lateral strain to linear strain will be a constant within elastic limit. This ratio is known as Poisson's ratio.	2	
2.	The $\perp^r$ distance at which the whole mass or area of the body is concentrated to the reference axis.	2	
4.	It is the turning moment or twisting moment of a force. It is the product of force and the distance from the point of application of force and the centre of shaft	2	
5.	Moment of resistance is the moment of the couple formed by the compressive force and tensile force due to bending or external bending moment.	2	10
	Part B		
II 1.	$R_B \times 4 = 2 \times 4 \times 2 + 10 \times 2$ $\therefore R_B = 9\,kN$ $\therefore R_A = 18 - 9 = \underline{9\,kN}$ 	moment eqn - 2 R_A - 2 R_B - 2 6	

Scoring Indicators

Code : **Version:**

Qn. No.	Scoring Indicators	Split score	Total score
2.	Sphere $V_1 = \frac{2\pi r^3}{3} = \frac{2\pi \times 30^3}{3} = 18000\pi\ mm^3$. $y_1 = 40 + \frac{3}{8}r = 40 + \frac{3}{8} \times 30 = 51.25\ mm$. Cone $V_2 = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi \times 30^2 \times 40 = 12000\pi\ mm^3$. $y_2 = \cancel{\frac{3\times4}{4}} = \cancel{10\,mm}\ \frac{3}{4} \times 40 = 30\,mm$. $\therefore \bar{y} = \frac{V_1 y_1 + V_2 y_2}{V_1 + V_2} = \frac{18000\pi \times 51.25 + 12000\pi \times 30}{30000\pi}$ $= \underline{42.75\ mm}$ from the top C.	eqn - 2 y_1, y_2 - 2 $\bar{y}$ - 2 6	
3.	$P = 20kN,\ l = 1m,\ A = 200\,mm^2,\ E = 100\ GPa$ $\delta l = \frac{Pl}{AE} = \frac{20\times10^3 \times 1000}{200 \times 100 \times 10^3} = \underline{1\,mm}$	eqn - 2 δl = 4 6	
4.	Volumetric strain: When the body is subjected to some forces, the ratio of change in volume to the original volume is called volumetric strain. Bulk modulus: The ratio of direct stress to volumetric strain, when a body is subjected to three mutually ⊥r stresses of equal intensity. Modulus of rigidity: Within elastic limit, the ratio of shear stress to shear strain is a constant. This constant is known as modulus of rigidity.	2 2 2 6	

1	2		
5.	$T_{ave} = 2255\,Nm.$, $D = 80mm.$ Max. Torque $= 1.4 \times 2255 = 3157\,Nm.$ $T = \frac{\pi}{16}\sigma_s D^3$ $\therefore \sigma_s = \frac{16T}{\pi D^3} = \frac{16 \times 3157 \times 10^3}{\pi \times 80^3} = \underline{31.4\,N/mm^2}$		eqn - 2 σ_s - 4 6
6.	Longitudinal stress:- It is the stress induced in the material of the thin cylinder in a direction parallel to the length of the shell. $\sigma_l = \frac{Pd}{4t}$, where P = pressure of internal fluid d = diameter of the cylinder. t = thickness of the cylinder. when this stress exceeds the permissible tensile stress of material of cylinder, the cylinder will burst into two cylinders.	3	
	Hoop or circumferential stress:- It is the tensile stress induced in the material of thin cylinder containing some fluid under pressure, in a direction tangential to the perimeter of the cylinder. If the stress exceeds the permissible stress, the cylinder will burst into two halves.	3 6	
7.	1) The material of the beam is homogeneous and isotropic 2) The value of young's modulus of elasticity is same in tension and compression. 3) The transverse sections which were plane		

2	3	4

remains plane after bending.

initially straight and all longitudinal

...nd into circular arcs with a

...e of curvature.

5) The radius of curvature is large compared with the dimensions of cross section.

6) Each layer of the beam is free to expand or contract, independently of the layer above or below it. $6\times1=6$ 42

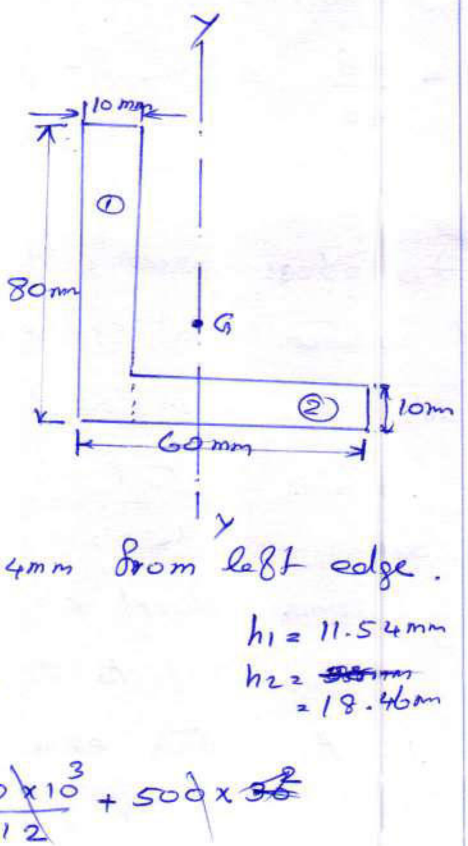
Part C

III (a) $a_1 = 80\times10 = 800\,mm^2$

$x_1 = 5\,mm$

$a_2 = 50\times10 = 500\,mm^2$

$x_2 = 35\,mm$



$$\bar{x} = \frac{a_1x_1 + a_2x_2}{a_1 + a_2}$$

$$= \frac{800\times5 + 500\times35}{1300} = \underline{16.54mm} \text{ from left edge.}$$

$h_1 = 11.54mm$

$h_2 = \cancel{35mm} = 18.46mm$

$$I_{yy} = I_{G1} + a_1h_1^2 + I_{G2} + a_2h_2^2$$

$$= \frac{10\times80^3}{12} + 800\times11.54^2 + \frac{50\times10^3}{12} + 500\times\cancel{30}^2$$

$$= \frac{80\times10^3}{12} + 800\times11.54^2 + \frac{10\times50^3}{12} + 500\times18.46^2$$

$$= 113203.95 + 274552.47$$

$$= \underline{\underline{387756.42\,mm^4}}$$

$\bar{x} = 3$

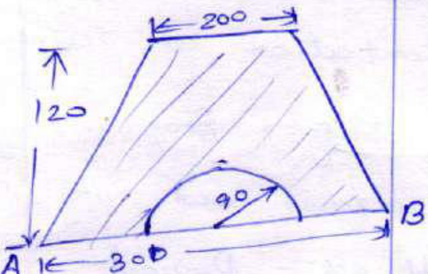
eqn - 2

$I_{yy} - 3$

8

1	2	3	4.

b) $a_1 = \frac{(200+300)}{2} 120 = 30000 \ mm^2$



$Y_1 = \frac{h}{3}\left(\frac{b+2a}{b+a}\right) = \frac{120}{3}\left[\frac{300+2\times 200}{300+200}\right]$

$= 56 \ mm.$

$a_2 = \frac{1}{2}\pi r^2 = \frac{1}{2}\pi \times 90^2 = 12723.45 \ mm^2$

$Y_2 = \frac{4r}{3\pi} = \frac{4\times 90}{3\pi} = 38.2 \ mm.$

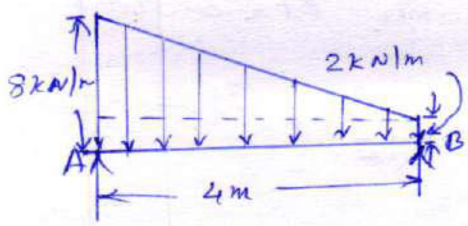
$\bar{y} = \frac{a_1 y_1 - a_2 y_2}{a_1 - a_2} = \frac{30000\times 56 - 12723.45\times 38.2}{30000 - 12723.45}$

$= \underline{\underline{69.1 \ mm}}$ from AB. , $\bar{x} = 150$ mm from A

eqn-2
$\bar{y}$ - 5
7 — 15

IV(a)

$R_A + R_B = \frac{(8+2)}{2} 4 = 20 \ kN$



$R_B \times 4 = 2\times 4\times 2 + \frac{1}{2}\times 8\times 4\times \frac{4}{3}$

$R_B = \underline{\underline{9.33 \ kN}}$

$R_A = 20 - 9.33 = \underline{\underline{10.67 \ kN}}$

moment-4
R_A - 2
R_B - 2
8

(b) $I_{zz} = I_{xx} + I_{yy}$ $\quad b = 200$ mm, $d = 300$ mm.

$= \frac{bd^3}{12} + \frac{db^3}{12}$

$= \frac{200\times 300^3}{12} + \frac{300\times 200^3}{12}$

$= 450\times 10^6 + 200\times 10^6 = \underline{\underline{650\times 10^6 \ mm^4}}$

I_{xx} - 2
I_{yy} - 2
I_{zz} - 3
7 — 15

V (a)(i) Elasticity: It is the property of a material by which the ~~bod~~ the deformation due to an external load is disappear on removal of the load. — 2

ii) Hardness :- It is the property of a material to resist indentation or surface abrasion — 2

iii) Ductility :- Property by which the material can be drawn out into thin wires. — 2

iv) Stiffness :- Property of resistance to bending — 2

8

b) $\sigma = 76.5\ Mpa$, $E = 90\ GPa$, $\alpha = 17\times10^{-6}/^\circ C$

$\sigma = \alpha t E$ — eqn - 2

$\therefore t = \dfrac{\sigma}{\alpha E} = \dfrac{76.5\times10^{6}}{17\times 90\times10^{3}} = \underline{\underline{50^\circ C}}$. — t - 5

7 — 15

VI(a) $A = 50\times50 = 2500\ mm^2$, $P = 500\ kN$, $\delta l = 0.5\ mm$,

$l = 200\ mm$, $\delta t = 0.04\ mm$

$\delta l = \dfrac{Pl}{AE}$, $\therefore E = \cancel{\delta l \cdot A}\ \dfrac{Pl}{A\cdot\delta l}$.

$= \dfrac{500\times10^{3}\times200}{2500\times0.5} = \underline{\underline{80\times10^{3}\ N/mm^2}}$. — E = 4

$\frac{1}{m}$ = 4

8

$\frac{1}{m} = \dfrac{\delta t/t}{\delta l/l} = \dfrac{0.04/50}{0.5/200} = \underline{\underline{0.32}}$

b) $A = 500\times200 = 100000\ mm^2$, $l = 2m$, $P = 150\ kN$, ~~E=200~~ $E = 200\ kN/mm^2$.

stress at elastic limit = $200\ N/mm^2$.

i) Strain energy $= \dfrac{\sigma^2}{2E}\times V = \dfrac{\left(\dfrac{150}{10^5}\right)^2}{2\times200}\times10^5\times2000 = \underline{\underline{1.125\ kNmm}}$ — 2

ii) Proof resilience $= \dfrac{\sigma^2}{2E}\times V = \dfrac{0.2^2\times10^5\times2000}{2\times200} = \underline{\underline{20000\ kNmm}}$. — 3

iii) Modulus of resilience $= \dfrac{\text{Proof resilience}}{V} = \underline{\underline{0.1\ N/mm^2}}$ — 2

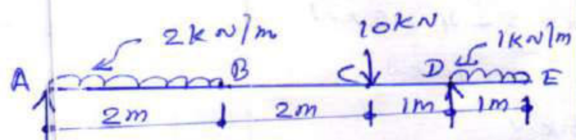
7 — 15

1	2	3	4

VII(a)

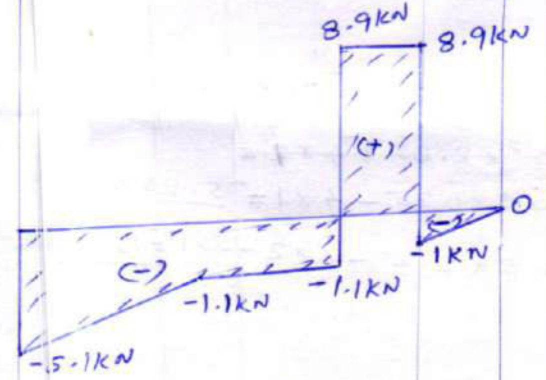
$R_D \times 5 = 1 \times 1 \times 5.5 + 10 \times 4 + 2 \times 2 \times 1$

$\therefore R_D = \dfrac{49.5}{5} = \underline{9.9 kN}$



$R_A + R_D = 15$

$\therefore R_A = 15 - 9.9 = \underline{5.1 kN}$



SFD

SF

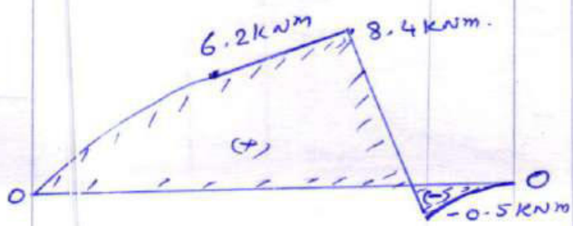
at $E = 0$

" $D = 0 - 1 = -1 kN + 9.9 = \underline{8.9 kN}$

" $C = 8.9 - 10 = \underline{-1.1 kN}$

" $B = -1.1 kN$

" $A = -1.1 - 4 = -5.1 kN$



BMD

BM

at $E = \underline{0}$.

" $D = -1 \times 1 \times 0.5 = \underline{-0.5 kNm}$.

" $C = -1 \times 1 \times 1.5 + 9.9 \times 1$
$= \underline{8.4 kNm}$.

" $B = 5.1 \times 2 - 2 \times 2 \times 1$
$= \underline{6.2 kNm}$

" $A = 0$.

Max +ve BM $= \underline{\underline{8.4 kNm}}$. — SFD 4, BMD - 4, $\underline{}$ 8

Max -ve BM $= \underline{\underline{-0.5 kNm}}$.

b) $D = 80 mm$, $\theta = 1.5^\circ$, $l = 5 m$, $\sigma_s = 42 MPa$, $N = 84 GPa$

$T = \dfrac{\pi}{16} \sigma_s D^3 = \dfrac{\pi}{16} \times 42 \times 80^3 = \underline{4222.3 \times 10^3 Nmm}$ — 3

T is also equal to $\dfrac{N\theta \times J}{l}$, $J = \dfrac{\pi D^4}{32} = \dfrac{\pi \times 80^4}{32} = 50265.48$

$\therefore T = \dfrac{84 \times 10^3 \times 50265.48 \times \frac{\pi}{180} \times 1.5}{5000} = \underline{1768.63 \times 10^3 Nmm}$ — 3

$\therefore$ Maximum torque that can be applied = lesser value

$= 1768.63 \times 10^3 Nmm = \underline{\underline{1768.63 Nm}}$ — 1, $\underline{}$ 7 — 15

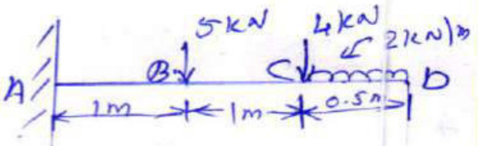
VIII (a) SF

at $D = 0$

" $C = 2\times0.5+4 = 5\,kN$

" $B = 5+5 = 10\,kN$

" $A = 10\,kN$



BM

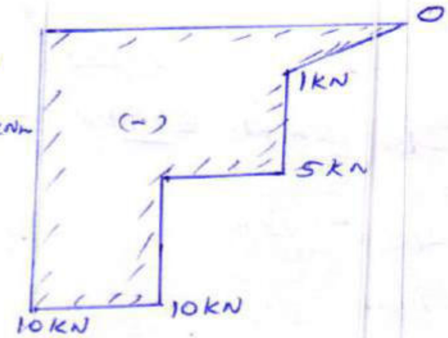
at $D = 0$

" $C = 2\times0.5\times0.25 = \bar{0}.25\,kNm$

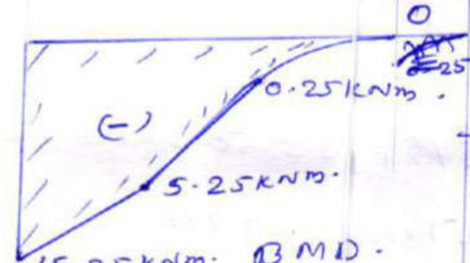
~~" $B = 2\times0.5\times2.25 - 4\times1 =$~~

at $B = -2\times0.5\times1.25 - 4\times1 = -5.25\,kNm$

" $A = -2\times0.5\times2.25 - 4\times2 - 5\times1 = -15.25\,kNm$



SFD — 4



BMD. — 4

8

b) $d = 2m,\ t = 10mm,\ P = 1.6\,MPa,\ E = 200\,GPa,\ \frac{1}{m} = 0.3$

Change in diameter, $\delta d = \frac{Pd^2}{4tE}\left[1-\frac{1}{m}\right] = \frac{1.6\times2000^2\,[1-0.3]}{4\times10\times200\times10^3}$

$= \underline{\underline{0.56\,mm}}$ — 3

Change in volume, $\delta V = \frac{\pi P d^4}{8tE}\left[1-\frac{1}{m}\right]$

$= \frac{\pi\times1.6\times2000^4\,(1-0.3)}{8\times10\times200\times10^3}$

$= \underline{\underline{3518.583\times10^3\,mm^3}}$ — 4

7   15

IX (a) $b = 200\,mm,\ d = 400\,mm,\ w = 10\,kN/m,\ \sigma_{max} = 50\,N/mm^2$

$M = \frac{wl^2}{8} = \frac{10\times l^2}{8} = 1.25\,l^2$

$$I = \frac{bd^3}{12} = \frac{200 \times 400^3}{12} = 1066.67 \times 10^6 \, mm^4$$

$$Z = \frac{I}{y_{max}} = \frac{1066.67 \times 10^6}{200} = 5.333 \times 10^6 \, mm^3.$$

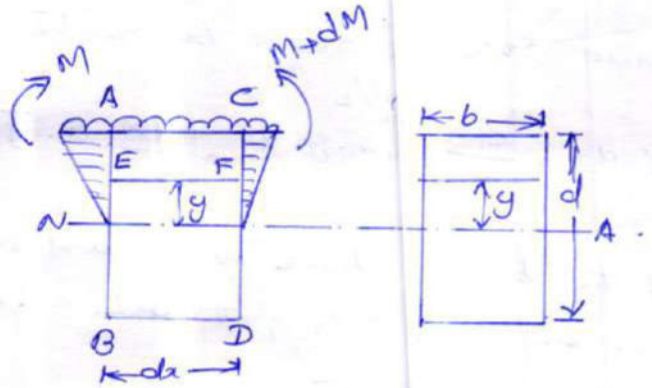
$$M = \sigma_{max} \times Z = \frac{wl^2}{8} = 1.25 l^2$$

$$\therefore l^2 = \frac{50 \times 5.333 \times 10^6}{1.25 \times 10^6} = 213.3$$

$$\therefore l = \underline{\underline{14.6 \, m}}.$$

8.

b)



Consider a small part of beam ABCD of length dx is loaded with a u.d.l as shown in fig.

Let M = BM at Section AB
M+dM = BM at section CD
F = Shear force at AB,
F+dF = shear force at CD
I = Moment of Inertia of the Section about its N.A.

consider an elementary strip at a distance 'y' from N.A.

Let σ = intensity of bending stress across AB at a distance y from N.A.

a = area of strip

$\frac{M}{I} = \frac{\sigma}{y}$ or $\sigma = \frac{M \cdot y}{I}$

$\therefore \sigma + d\sigma = (M + dM) \times \frac{y}{I}$

$\therefore$ the force across AB = stress × area = $\sigma \times a$

~~and the force across~~ $= \frac{M}{I} \times y \times a.$

the force across CD = $(\sigma + d\sigma)\, a = \frac{(M + dM)}{I} \times y \times a.$

$\therefore$ Net unbalanced force $= \frac{(M + dM)}{I} \times y \times a - \frac{M}{I} \times y \times a$

$$= \frac{dM}{I} \times y \times a.$$

The total unbalanced force (F) above the neutral axis $= \int_0^{d/2} \frac{dM}{I} \cdot a \cdot y \cdot dy = \frac{dM}{I} \int_0^{d/2} a \cdot y \cdot dy = \frac{dM}{I} \cdot a \cdot \bar{y}.$

where, A = area of the beam above N.A and
$\bar{y}$ - distance between C.G of the area and N.A.

We know that $\tau = \frac{Force}{Area} = \frac{\frac{dM}{I} \cdot A\bar{y}}{dx \cdot b} = \frac{dM}{dx} \cdot \frac{A\bar{y}}{Ib}$

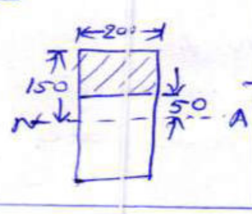
$$= \frac{F \cdot A\bar{y}}{Ib} \qquad \left(\because \frac{dM}{dx} = F \right)$$ 7 15

x(b) $b = 200mm,\ d = 300mm,\ F = 100kN.$

$\tau_{max} = 1.5\ \tau_{av} = 1.5 \times \frac{F}{bd} = \frac{1.5 \times 100 \times 10^3}{200 \times 300} = 2.5\ N/mm^2.$ --- 3

τ at $y = 50mm$, $\tau = \frac{F \cdot A\bar{y}}{Ib} = \frac{100 \times 10^3 \times 200 \times 100 \times 100}{\frac{200 \times 300^3}{12} \times 200}$

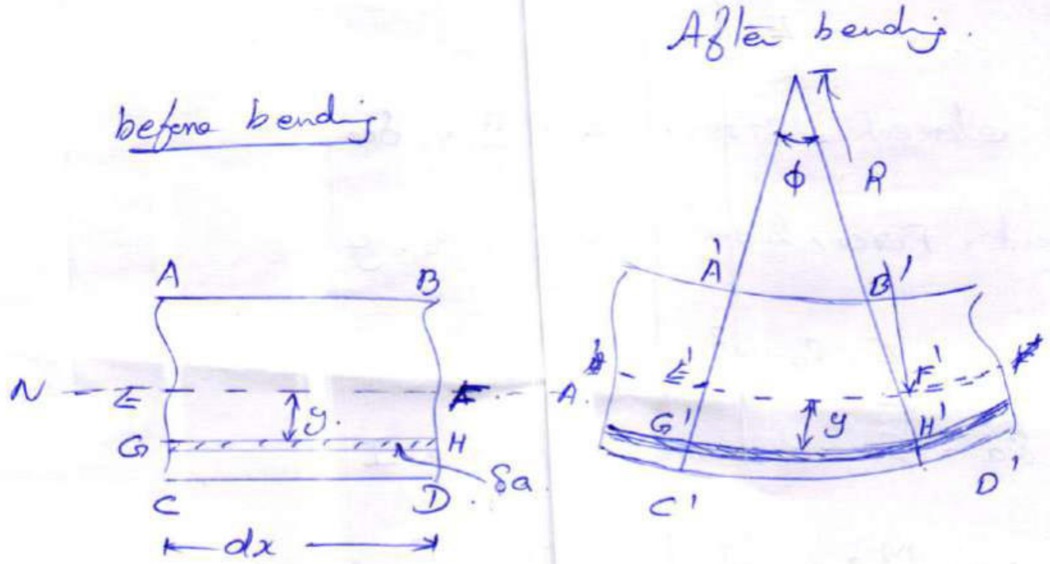
$= 2.22\ N/mm^2$



--- 4

7

x (a).



Let R be the radius of curvature

ϕ be the angle subtended by the cross sections

$A'C'$ and $B'D'$ at the centre of curvature.

dx = length of beam. between sections AC & BD.

$$EF = E'F' = R.\phi.$$

$$\text{Strain} = \frac{\text{Change in length}}{\text{Original length}} = \frac{G'H' - GH}{GH}.$$

$$GH = EF = R\phi.$$

$$G'H' = (R+y)\phi$$

$$G'H' - GH = (R+y)\phi - R\phi$$

$$\therefore \frac{G'H'}{GH} = \frac{(R+y)\phi - R\phi}{R\phi} = \frac{y\phi}{R\phi} = \frac{y}{R} \quad --(1)$$

$$\text{Strain} = \frac{\text{Stress}}{E} = \frac{\sigma}{E} \quad --(2)$$

$$\therefore \frac{y}{R} = \frac{\sigma}{E} \quad \text{or} \quad \frac{\sigma}{y} = \frac{E}{R} \quad ---(3)$$

$$\therefore \sigma = \frac{E}{R} . y .$$

Force on element = stress × area = $\frac{E}{R} . y . \delta a$.

$\therefore$ Moment = Force × lever arm = $\frac{E}{R} . y . \delta a . y$.

or $M = \frac{E}{R} . \delta a . y^2$.

$y^2 . \delta a$ = Second moment of area = I .

$$\therefore M = \frac{E}{R} . I \quad \text{or} \quad \frac{M}{I} = \frac{E}{R} . \quad \ldots (4)$$

from eqns (3) and (4)

$$\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R} .$$ - bending eqn .