SCHEME OF VALUATION

(Scoring Indicators)

REVISION : 2010 — COURSE CODE: 3021
THEORY OF STRUCTURES -I

Q.NO.	SCORING INDICATOR	SPLIT UP SCORE	SUB TOTAL	TOTAL
I.	**PART -A**			
1.	***Centre of gravity:*** - the centre of gravity of a body is that point located inside the body through which the whole weight of the body acts. ***Centroid:*** - this is a point, where the whole area of the lamina is assumed to be concentrated.	1 1		2
2.	It is the maximum amount of strain energy that can be stored in an elastic body such that its elastic limit is not exceeded. Ie., without causing any permanent set or permanent deformation.	2		2
3.	A member is said to be under pure torsion when it is subjected to torque only, without associated with any bending moment or axial force.	2		2
4	When a thin – walled cylinder is subjected to internal pressure, three mutually perpendicular principal stresses will be set up in the cylinder materials, namely • Circumferential or Hoop stress • Radial stress • Longitudinal stress **Circumferential or Hoop stress:** This is the stress which is set up in resisting the bursting effect of the applied internal pressure and can be most conveniently treated by considering the equilibrium of the cylinder. The hoop stress is the force exerted circumferentially (perpendicular both to the axis and to the radius of the object) in both directions on every particle in the cylinder wall.	1 1		2
5	Any member subjected to compressive force is called ***Strut***. When a strut is vertical in position it is called ***Column***.	1 1		2
II	**PART - B**			
1	To locate the C.G. of the whole section. Let XX be the axis through the centroid G and parallel to the base. Let the distance of the C.G. of the section from the base = Y $Y = \frac{(100 \times 40) \times 20 + (130 \times 20) \times 105 + (60 \times 30) \times 185}{100 \times 40 + 130 \times 20 + 60 \times 30}$ = **81.67 mm** from the base. To determine the M.I. of the whole section about the base	2		

	$I_{base} = \frac{(100 \times 40^3)}{12} + (100 \times 40) \times 20^2 + \frac{(20 \times 130^3)}{12} + (20 \times 30) \times 105^2 + \frac{(60 \times 30^3)}{12} + (60 \times 30) \times 185^2$ $= \underline{\mathbf{96.2 \times 10^6\ mm^4}}$ Let M.I. of the whole section about an axis through the C.G. parallel to the base = I xx $I_{base} = I_{xx} + A\,Y^2$ [Where A = area of the whole section] $I_{xx} = I_{base} - A\,Y^2$ $= 96.2 \times 10^6 - (100 \times 40 + 130 \times 20 + 60 \times 30) \times 81.67^2$ $= \underline{\mathbf{40.17 \times 10^6 mm^4}}$			
2	In Industries, there may be situations where a member is subjected to external axial forces not only at its ends, but also at some of its interior cross-sections along the length of the body. In such cases, the total deformation of the member may be evaluated by splitting up the entire member (ie., free body diagram) into several sections and summing up the deformation of each of these sections. This principle of finding out the resultant deformation is known as *principle of superposition.* ***Hooke's*** law states, "Whenever a material is loaded within its elastic limit, the developed stress at any point within the material is proportional to the corresponding strain".	3 3		6
3	a) ***Shear force*** at a section is the resultant vertical force either to the right or to the left of the section. b) ***Bending moment*** at a section is the algebraic sum of the moments of all forces either to the left or the right of the section. c) The point where the B.M. diagram cuts the base after changing its sign from positive to negative or vice-versa, is termed as the ***point of contraflexure*** or point of inflexion. At this point, the bending of the beam changes from sagging to hogging or vice-versa and therefore, the B.M. is zero.	2 2 2		6
4	(i) It is the ratio of the effective length of the column (***L***) to the least radius of gyration of its cross-section (***K***) ie., the ratio of L/K is known as *slenderness ratio.* ((ii)) In case of long column, the length actually involved in bending is called its ***equivalent length or effective length***. If '*l*' is actual length of a column, then its equivalent length or effective length '*L*' may be obtained by multiplying it with some constant factor *C*, which depends on the end fixation of the column, ie., ***L = C X l.***	3 3		6

	Values of the factor C for different end conditions are taken.			
5	Given: Diameter of shaft, D = 100 mm Allowable shear stress in the shaft, $f_s = 100\ N/mm^2$ Let T = safe torque that can be transmitted through the shaft. J = Polar moment of inertia of the cross-section of the shaft. R = Radius of the shaft = D/2 = 100 / 2 = 50 mm $J = {}^{\pi D4}/_{32} = {}^{\pi \times 100\ 4}/_{32} = \underline{9.817 \times 106\ mm^4}$ From Torsion equation, $T / J = f_s / R$ $T = f_s \times {}^{J}/_{R} = 100 \times 9.817 \times 10^{6} / 50$ $= 19.63 \times 10^6$ Nmm = <u>19.63 KNm.</u>	2 2 2		6
6	(i) ***Elasticity*** is the property of a material which undergoes deformation when subjected to external force up to a certain limit such that deformation disappears on removal of the force. (ii) ***Plasticity*** is the property of a material which undergoes deformation when subjected to external force such that the deformation does not disappear at all, even after the force in removed. (iii) ***Ductility*** is the property of a material which allows itself to be drawn out by tension to a smaller section. During ductile extension, a material generally shows a certain degree of elasticity together with a considerable amount of plasticity.	2 2 2		6
7	If n = number of a members in a frame or truss, and j = number of joints Then an equation, (n = 2j – 3) determines the type of structure. (i) If n = 2j – 3, this equation is satisfied, then the given truss is a Perfect frame (efficient frame) (ii) If n < 2j – 3, the frame is an Imperfect (deficient frame). (iv)If n > 2j – 3, the frame is a redundant frame	2 2 2		6

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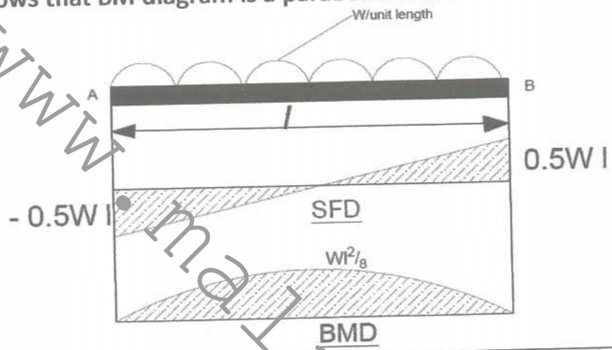
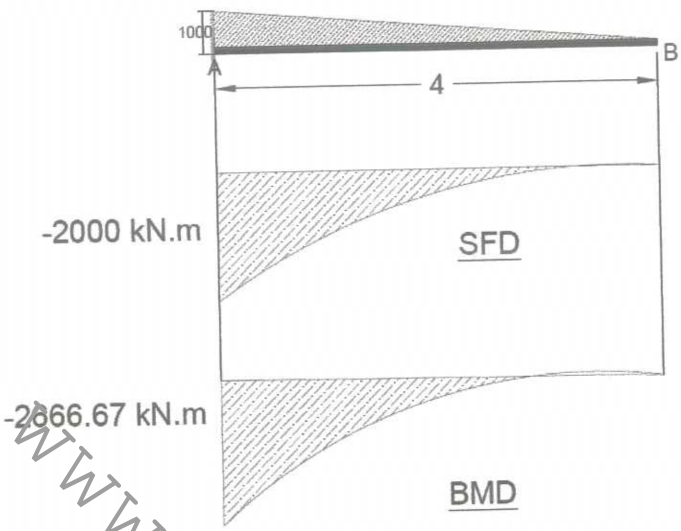
III	**PART -C**				
(a)	1. **Static friction:-** if the two surfaces which are in contact are at rest, the friction experienced is called static friction or in other wards it is the friction when the body tends to move. **2. Dynamic friction**: - if one of the surfaces starts moving and the other is at rest, the opposing force which acts in the opposite direction of the movement of the surface is called dynamic friction. It is also called kinetic friction. Dynamic friction is of the following two types. *1. **Sliding friction*** – it is the friction developed in a body when it slides over another body. 2. ***Rolling friction*** – it is the friction developed in a body when it rolls over another body.	2 2 1 1			6
(b)	Weight of the body, W = 500 N Force applied, P = 350 N Inclination, $\alpha = 30^0$ Let μ = coefficient of friction R = normal reaction F = force of friction = μR The body is in equilibrium under the action of the forces as shown in figure. Resolving the forces along the plane 500 Sin 30 + F = 350 500 Sin 30 + μR = 350 Resolving the forces normal to the plane R = 500 Cos 30 = 433 N Substituting the value of R in eq.(1) 500 sin 30 + μ x 433 = 350 433 μ = 350 – 500 Sin30 Coefficient of friction, μ = 100/433 = **0.23**	2 2 2 2 1			9
IV					
(a)	Span of the beam = 9m Load at the end A = 0 Load at the end B = 900N/m Total load on the beam = wl = (0 + 900) / 2 X 9 = 4050 N This load will be acting at the C.G. of the triangle ABC. Consider the equilibrium of the beam,	2			

	$\Sigma V = 0$; $\Sigma M = 0$ $R_A + R_B = 4050$ .... (1) Taking moment about A, $R_B \times 9 = (4050 \times 2/3 \times 9) = 0$ $R_B = 2700$ N. $R_A = 1350$ N.	2 1 1		6	
(b)	The body is symmetrical about y-y axis. Therefore its C.G. will lie on this axis. The bottom of the hemisphere © be the point of reference. Hemisphere :- volume, $v_1 = {}^2/_3 \pi r^3 = {}^2/_3 \pi\, 30^3 = \mathbf{18{,}000\ \pi\ mm^3}$ $Y_1 = {}^5/_8 r = {}^5/_8 \times 30 = 18.75$ mm Right circular cone:- volume, $v_2 = {}^1/_3 \pi r^2 h = {}^1/_3 \pi\, 30^3 \times 40$ $= \mathbf{12{,}000\ \pi\ mm^3}$ $Y_2 = 30 + 40/4 = 40$ mm Distance between the C.G. of the body and bottom of the hemisphere C, $\bar{y} = \frac{v_1 y_1 + v_2 y_2}{v_1 + v_2} = \frac{18000\pi \times 18.75 + 12000\pi \times 40}{18000\pi + 12000\pi}$ $= \mathbf{27.25\ mm}$	 3 3 2 1		9	
V	**MODULE - II**				
(a)	Given; $P = 20$ kN $= 20 \times 10^3$ N $L = 2.5$ m $= 2500$ mm $A = 1000 mm^2$ $E = 200$ GPa. $= 200 \times 10^3$ N/mm^2 Stress in the rod when the load is suddenly applied, $\sigma = 2 \times {}^P/_A$ $= 2\,(20 \times 10^3) / 1000 = \mathbf{40\ N/mm^2}$ Volume of the rod $= 2.5 \times 10^3 \times 10^3 = \mathbf{2.5 \times 10^6\ mm^3}$ Strain energy absorbed in the rod, $U = (\sigma^2 / 2E)\, V$ $= 40^2 / (2 \times 200 \times 10^3) \times 2.5 \times 10^6$ $= \mathbf{10\ kN\text{-}mm.}$	 2 2 2 1		7	
(b)	Given; Length of the rod = L = 4m =400 cm Diameter of the rod , d = 20 mm = 2 cm Decrease in temperature, $T = 60^0 - 20^0 = 40\ ^0C$ $\alpha_s = 12 \times 10^{-6} / {}^0C$ $E_s = 2 \times 10^6$ kg/cm^2 Due decrease in length, $\delta L = L\, \alpha_s\, T$ $= 400 \times 12 \times 10^{-6} \times 40 = \mathbf{0.192\ cm.}$ **(a) When the plates do not yield** Contraction prevented = 0.192 cm Strain in the rod $e = \delta L / L = 0.192 / 400 = 0.00048$ Temperature stress induced in the rod, $f = e \times E_s$ $= 0.00048 \times 2 \times 10^6$ $= \mathbf{960\ kg/cm^2}$ Force exerted by the rod = Stress X sectional area of the rod	 1 2			

	$= 960 \times (\pi / 4) \times 2^2$ $=$ **3015.93 kg(Tensile)**	2		
	(b) If yield is 1 mm Then decrease in length actually prevented $= 0.192 - 0.100 = 0.092$ cm Strain actually prevented, $e = 0.092 / 400 = 2.3 \times 10^{-4}$ Temperature stress induced in the rod $= e \times E$ $= 2.3 \times 10^{-4} \times 2 \times 10^6$ $=$ **460 kg / cm²** Force exerted by the rod $= 460 \times (\pi / 4) \times 2^2 =$ **1445.13 kg.**	1 1 1		8
VI				
(a)	Given : $P = 50$ kN, Area $= 100$ mm $\times$ 200 mm, $E = 200$ kN/mm², $l = 4$m $= 4000$mm (i) Stress setup in the bar material, $\sigma = P / A$ $= 50/100 \times 200 =$ **2.5X10⁻³ kN / mm²** (ii) Strain produced, $\epsilon = \sigma / E$ $= 2.5 \times 10^{-3} / 200 =$ **1.25 X 10⁻⁵** (iii) Elongation of the bar, $\delta l = Pl / AE$ $= 50 \times 4000 / 200 \times 100 \times 200 =$ **0.05 mm** (iv) Work done = Average force X deformation $= {}^{(0+P)}/_2 \times \delta l = {}^{(0+50)}/_2 \times 0.05 =$ **1.25 kN-mm.**	2 2 2 2	8	
(b)	**(i) Bulk modulus(K):-** when a body is subjected to three mutually perpendicular stresses of equal intensity, the ratio of direct stress to the corresponding volumetric strain is known as bulk modulus. It is usually denoted by K. $K = \text{Direct stress / Volumetric strain} = \frac{\sigma}{\delta V / V}$ **(ii) Volumetric strain(ϵV):-** whenever a material body is subjected to forces, it undergoes change in dimensions in all directions. Hence the body undergoes change in volume. Volumetric strain is the ratio of the change in volume to the original volume of a material body. $\epsilon V = \delta V / V$ **(iii) Poisson's ratio ($1/m$ or μ):-** If a body is stressed within its elastic limit the ratio of the lateral strain to longitudinal strain is constant for a given material. This constant is called poisons ratio. $\frac{1}{m} = \frac{Lateral\ strain}{Longitudinal\ strain}$	3 2 2		7
VII				
(a)	$R_A + R_B = Wl/2$			

		Shear force at any section X at a distance x from B, $SF_x = {}^{Wl}/_2 - Wx = 0.5\,Wl - Wx$ $SF_B = 0.5\,wl$; $SF_A = -0.5\,Wl$ $SF_C = Wl/2 - Wl/2 = 0$ This shows that SF is zero at mid point and 0.5Wl at B and decreases uniformly by a straight line law to -0.5Wl at A. $BM_A = 0$; $BM_B = 0$ $BM_C = {}^{Wl}/_2\ X({}^{l}/_2) - {}^{Wl}/_2 \times {}^{l}/_4 = Wl^2/_8$ (Maximum BM where SF changes sign) $BM_X = Wl/_2 \,.\, X - WX \,.X/_2 = Wlx/_2 - Wx^2/_2$ This shows that BM diagram is a parabolic curve. 	1 1 1 2 2		7
	(b)				
VIII					
	(a)	Method of finding torsional equation for hollow circular shaft is same as that of solid circular shaft. Torsion equation. $T/J = \sigma_s/R = N\theta/l$ Where R = Outer radius of the hollow shaft $T = \frac{\pi\sigma_s}{16}[\frac{D^4-d^4}{D}]$ $J = \frac{\pi}{32}[D^4 - d^4]$ d = inner diameter of the hollow shaft D = outer diameter of the hollow shaft	3 2 2		7
	(b)	Shear force, $SF_B = 0$ $SF_A = -1000 \times 4 = -2000$kN.m Bending moment, $BM_B = 0$ $BM_A = -1000 \times 4/2 \times 4/3 = -2666.67$ kN.m	2 2		

			2 2		8
IX					
	(a)	A column or strut is never subjected to buckling load because if the axial compressive load on the column is greater than the buckling load, the column will buckle. In actual practice, the column is subjected to a load much less than the buckling load. This load is called working load or safe load of a column. Working load is obtained by dividing the buckling load by a number called factor of safety. Working load = $^{\text{buckling load}} / _{\text{factor of safety}}$	3 2		5
	(b)	(i) Find the reactions at B and C Taking moments about B, $\Sigma V = 0$, $\Sigma M = 0$ $R_B + R_C = 10$ R_c X 10 = 10 X AB Cos 60 = 10 X 5 Cos 60 = <u>2.5 KN.</u> AB = BC Cos 60 = 10 X ½ = 5 R_B = <u>7.5KN</u> By method of Joints, Consider the equilibrium of the joint C, then $\Sigma H = 0$, $\Sigma V = 0$ P_{CA} X Sin 30 = R_C P_{CA} = 2.5/Sin30 = <u>5KN.(Compression)</u> $P_{CB} = P_{CA}$ X Cos 30 = <u>4.33 KN (Tension)</u> Consider the equilibrium of the joint B, P_{BA} X Sin 60 = R_B P_{BA} = 7.5 / Sin 60 = <u>8.66KN (Compression)</u> $P_{CB} = P_{BA}$ X Cos 60 = 8.66 X Cos 60 = <u>4.33KN (Tension)</u>	2 2 2 2 2		10
X					
	(a)	Euler's formula gives correct results only for very long columns. But Rankine's formula is applicable for columns ranging from very long to short ones.			

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$$\frac{1}{P_R} = \frac{1}{P_C} + \frac{1}{P_E}$$

P_R = Crippling load by Rankine's Formula
$P_C = f_C.A$
= Ultimate crushing load for the column
$P_E = \frac{\pi^2 EI}{{l_e}^2}$
= Crippling load by Euler's formula

$$\frac{1}{P_R} = \frac{1}{P_C} + \frac{1}{P_E} = \frac{P_E + P_C}{P_C.P_E}$$

$$P_R = \frac{P_C.P_E}{P_C + P_E} = \frac{P_C}{1 + \frac{P_C}{P_E}}$$

$$P_R = \frac{f_C.A}{1 + f_C.A \times \frac{{l_e}^2}{\pi^2 E}} = \frac{f_C.A}{1 + \frac{f_C}{\pi^2 E} \times \frac{A{l_e}^2}{Ar^2}}$$

$$P_R = \frac{f_C.A}{1 + a\left(\frac{l_e}{r}\right)^2} = \frac{P_C}{1 + a\left(\frac{l_e}{r}\right)^2}$$

P_C = Crushing load of the column material
f_C = Crushing stress of the column material
A = Cross − sectional area of the column
a = Rankine's constant $\left(= \frac{f_C}{E\,\pi^2}\right)$
L_e = Equivalent length of the column,
r= Least radius of gyration

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<td>(b)</td>
<td>Given: l = 12 m = 1200 cm; A = 1m^2 = 10^4 cm^2; E = 2X 104 KN/cm^2
a) <u>Both ends of the column are pinned</u>
Buckling load, $P_E = \frac{\pi^2\ EL}{L^2}$ Where L = l</td>
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	$P_E = \dfrac{\pi^2 \ x\ 2x104x100x(100)3}{1200^2}$ = <u>1142310.8kN</u> <u>(b) one end fixed and other end free</u> Buckling load, $P_E = \dfrac{\pi^2\ EL}{L^2}$ Where L = 2l $\therefore P_E = \dfrac{\pi 2\ 2\ X\ 104\ (100x100)3/12}{(2\ X\ 1200)^2}$ = <u>285577.7 kN</u>	3 2 3		10

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