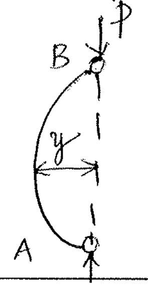
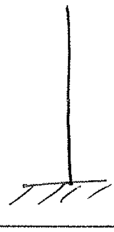
SCORING INDICATORS

Course Name: Theory of structures

Course Code: 3014 QID: 2110Q20089

Q No.	PART A Answer	Splitup	Total Mark
1.	Algebraic sum of all vertical forces to left or right of the section.	1	1
2.	$\frac{M}{I} = \frac{f}{y} = \frac{E}{R}$	1	1
3.	Length 'l' divided by least radius of gyration 'k'	1	1
4.	It is the angle that soil makes with horizontal surface with out slumping.	1	1
5.	$T/J = \tau/r = G\theta/L$	1	1
6.	Slope and deflection of a beam due to several loads is equal to sum of those due to individual loads.	1	1
7.	Moment required to produce unit rotation of end without translation. $k = \frac{EI}{L}$	1	1
8.	Continuous beams.	1	1
9.	Portal frame	1	1
			9

PART B

II			
1.	A beam is a structural member used to resist vertical loads, shear forces and bending moments. a) Simply supported beam b) Cantilever beam c) Overhanging beams d) Continuous beams e) Fixed beam. (Any two)	1 1 1	3
2.	a) The beam subjected to pure bending and is therefore free from shear force b) Material is isotropic and homogeneous c) A transverse section of the beam which is plane before bending will remain plane after bending.	1 1 1	3
3.	Both ends pinned  Both ends fixed  One end fixed Other end free 	1 each	3

4. A dam is liable to fail due to (i) Overturning (ii) Sliding (iii) due to tensile stress developed at the base (iv) due to excessive compressive stress. 1

Checks

(i) Check against overturning

The resultant of total weight of the dam 'W' and total water pressure 'P' must lie with the base. 1

Overturning moment $M_o = P.\frac{h}{3}$

Restoring moment $M_R = W(b - \bar{x})$

(ii) Check against sliding

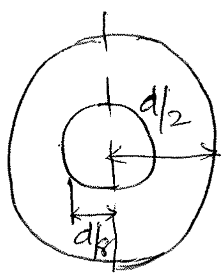
Maximum available frictional resistance should be greater than horizontal water pressure P 1

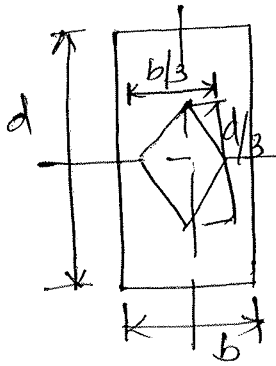
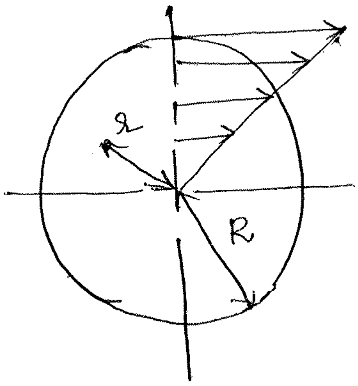
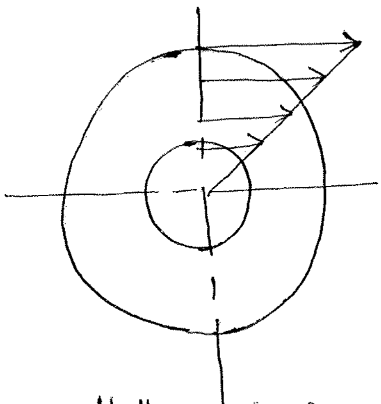
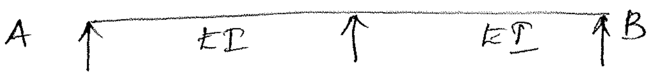
$\mu . W > P$

(iii) Check against excessive compressive stress.

Maximum compressive stress developed at the base should not exceed the permissible compressive stress of masonry or concrete with which dam is constructed 1

$f_{max} \leq f$

	Check for tensile stress. The section should be designed such that the resultant should be with in the middle third of the base ie, $e < \frac{b}{6}$.	1	3
5.	(Any three) 1) Bending moments are of lesser value in fixed beams.	1	
	2) Fixed beams have lesser value of max deflection.	1	
	3) The fixed beam is usually stronger and stiffer than the similar beam with simply supported conditions.	1	3
6.	Slope, $\theta = \frac{Wl^3}{6EI} = \frac{30\times(2000)^3}{6\times2\times10^5\times160\times10^6}$ $= 1.25\times10^{-3}$	1.5	
	Deflection, $\delta = \frac{Wl^3}{8EI} = \frac{30\times(2000)^4}{8\times2\times10^5\times160\times10^6}$ $= 1.875\,mm$	1.5	3
7.	 Area of core $= \pi e^2 \quad (e = \frac{d}{8})$ $= \frac{\pi d^2}{64}$	1.5	

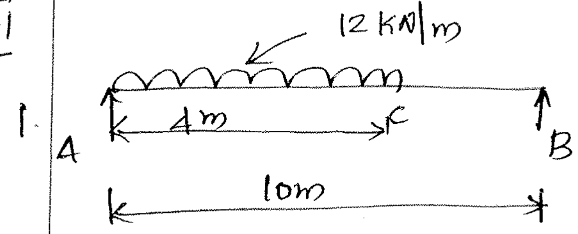
	rectangular section  $e_x = b/6$ (for x axis) $e_y = d/6$ (for y axis)	1.5	3
8.	 Solid circular shaft  Hollow circular shaft	2	
	Maximum shear stress occurs at the outer surface of the shaft where $r = R$	1	3
9.	$M_A L_1 + 2M_B(L_1+L_2) M_C L_2 + \frac{6A_1\bar{x}_1}{L_1} + \frac{6A_2\bar{x}_2}{L}$ $+ 6EI\left(\frac{\delta_1}{L} + \frac{\delta_2}{L}\right) = 0$ 		

	For no settlement $E_1 I_1 = E_2 I_2 = EI$ & $\delta_1 = \delta_2 = 0$ $M_A L_1 + 2M_B(L_1+L_2) + M_C L_2 + \frac{6A_1\bar{x}_1}{L_1} + \frac{6A_2\bar{x}_2}{L_2} = 0.$	1.5	3
10.	Distribution factor : The factor by which the applied moment is multiplied to obtain end moment of any member. OR Ratio of stiffness of that member to the total stiffness of all members meeting at the joint $\frac{K_1}{\Sigma K}$, $\Sigma K = K_1+K_2+K_3$... Carry over factor It is defined as the moment induced at the fixed end of a beam by the action of a moment applied at the other end which is hinged. OR Ratio of induced moment to applied moment	1.5 1.5	3

PART C

III

1.



$R_B \times 10 = 12 \times 4 \times \frac{4}{2}$ $\qquad$ $R_A = W - R_B$

$R_B = \frac{96}{10} = \underline{\underline{9.6\,kN}}$ $\qquad$ $= (12 \times 4) - 9.6$

$= \underline{\underline{38.4\,kN}}$ $\qquad$ 1

Consider any section x from A & C,
shear force at C is given by

$F_x = +R_A - 12x = 38.4 - \cancel{1}2x$

At $x = 0$,

$F_A = 38.4\,kN$ $\qquad$ 1

At $x = 4$

$F_C = 38.4 - 12 \times 4 = -9.6\,kN$

SF is 0 at x m from A,

$0 = R_A - 12x$

$= 38.4 - 12x$

$x = 3.2\,m$ $\qquad$ 1

BM

$$M_x = R_A \cdot x - 12 \cdot x \times \frac{x}{2}$$

$$= R_A \cdot x - 6x = 384x - 6x^2$$ 1

at $x = 0$,

$$M_A = 0$$

at $x = 4m$ $\quad M_C = 38.4 \times 4 - 6 \times 4^2$

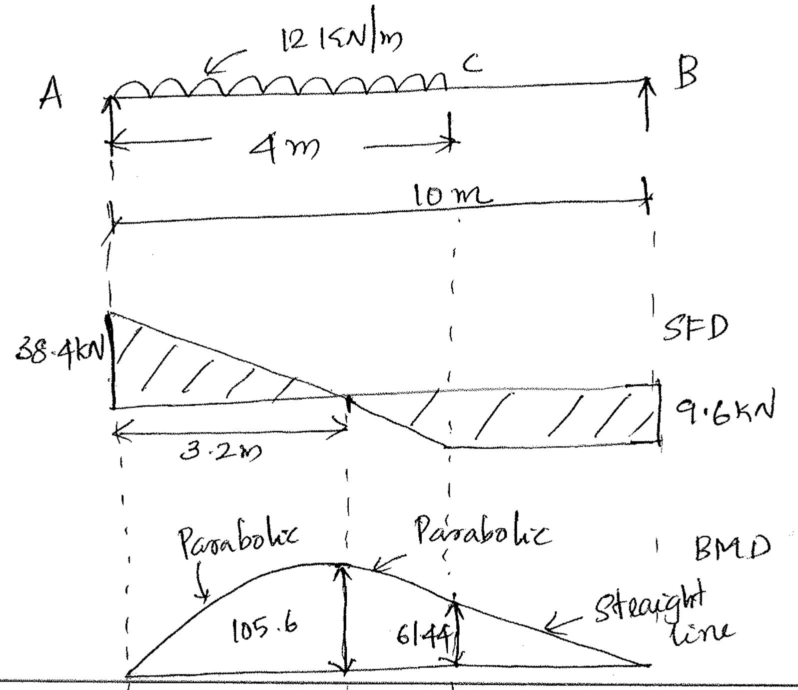
$$= \underline{\underline{105.6 \text{ KN-m}}}$$

at $x = 3.2m$

$$M_D = 38.4 \times 3.2 - 6 \times 3.2^2$$ 1

$$= 122.88 - 61.44$$

$$= \underline{\underline{61.44 \text{ KN-m}}}$$



1

1

OR

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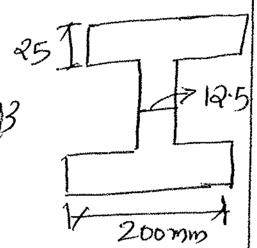
2. Moment of inertia I



$$= \frac{BD^3}{12} - \frac{(B-b)\,d^3}{12}$$

$$= \frac{200 \times 400^3}{12} - \frac{(200-12.5)\,350^3}{12} \qquad 2$$

$$I = \underline{\underline{396744792 \ mm^4}}$$

$$= \underline{\underline{39.67 \times 10^7 \ mm^4}}$$

$$\text{Area}, A = (25 \times 200) \times 2 + 12.5\,(400-50)$$

$$= 10{,}000 + 4375 \qquad 1$$

$$= \underline{\underline{14375 \ mm^2}}$$

$$\text{Avg stress} = \frac{F}{A} = \frac{250 \times 10^3}{14375} = \underline{\underline{17.39 \ N/mm^2}} \qquad 2$$

Maximum shear stress

$$= \frac{F}{Ib}\left(\frac{B}{8}\left(D^2-d^2\right) + \frac{bd^2}{8}\right)$$

$$= \frac{250 \times 10^3}{39.67 \times 10^7 \times 12.5}\left[\frac{200}{8}\left(400^2 - 350^2\right) + \frac{12.5 \times 350^2}{8}\right] \qquad 2$$

$$= \underline{\underline{56.908 \ N/mm^2}}$$

7

3.	<u>Assumptions</u> 1) Columns are initially straight 2) Material of column is homogeneous and isotropic 3) The columns carry perfectly axial loads. 4) The cross section of column is uniform through out. 5) The length of column is very large compared to lateral dimensions 6) The self weight of column is negligible 7) The column fails by buckling alone 8) Limit of proportionality is not exceeded 9) Pin joints are frictionless and fixed ends are fully rigid. 10) Shortening of column due to direct compression is neglected	1 each any 7	7
	OR		
4.	$H = 10m, \quad a = 1.6m, \quad b = 3.4m$ $\rho = 20\,kN/m^3 \quad \gamma = 10\,kN/m^3 \quad h = ?$ Water pressure $P = \frac{\gamma h^2}{2} = \frac{10h^2}{2} = 5h^2$ $W = \left(\frac{a+b}{2}\right) \cdot H\rho$	1 1	

$$= \left(\frac{1.6+3.4}{2}\right) \times 10 \times 20$$

$$= 500\ KN$$

Position where the resultant cuts the base is given by

$$z = \bar{x} + \frac{P}{W} \cdot \frac{h}{3}$$

$$\bar{x} = \frac{a^2+ab+b^2}{3(a+b)} = \frac{1.6^2+1.6 \times 3.4+3.4^2}{3(1.6+3.4)}$$

$$= \underline{\underline{1.304\ m}}$$

For no tension to develop top at base

$$z = \frac{2}{3} \times b$$

$$2/3 \times 3.4 = 1.304 + \frac{5h^2}{500} \times \frac{h}{3}$$

$$2.2667 = 1.304 + \frac{h^3}{300}$$

$$h^3 = (2.2667 - 1.304) \times 300$$

$$= \sqrt[3]{288.81} = \underline{\underline{6.61\ m}}$$

5. $$I = \frac{bd^3}{12} = \frac{250 \times 350^3}{12} = 89.32 \times 10^7\ mm^4$$ 2

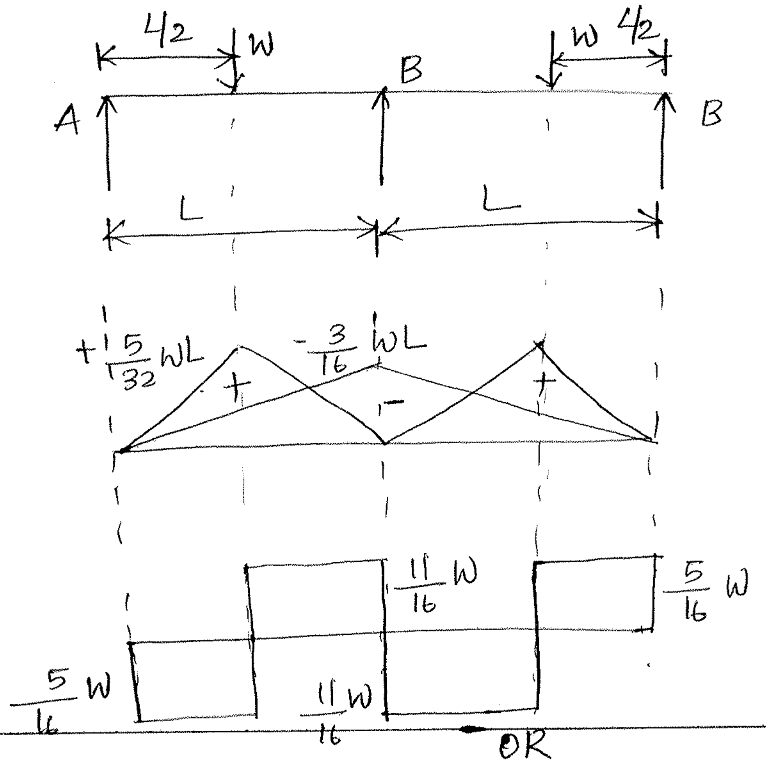
$$\theta = \frac{W l^3}{24EI} = \frac{60 \times (2000)^3}{24 \times 2 \times 10^5 \times 89.32 \times 10^7}$$ 2

$$= \underline{\underline{1.119 \times 10^{-4}}}$$

	deflection: $y_{max} = \frac{5}{384}\frac{wl^4}{EI}$	3	
	$= \frac{5}{384}\frac{60\times(2000)^4}{2\times10^5\times 89.32\times10^{7}}$		7
	$= \underline{\underline{0.0699\,mm}}$		
	OR		
6.	$P = 105\,kN$		
	$N = 160\,rpm$		
	$P = \frac{2\pi NT}{60}$		
	$T = \frac{60P}{2\pi N} = \frac{60\times105\times10^3}{2\pi\times160} = 6266.72\,N\text{-}m$	1	
	From $\frac{T}{J} = \frac{\tau}{R}$		
	$T = \frac{\pi}{16}\tau d^3$		
	$\frac{6266.72\times10^3\times16}{\pi\times65} = d^3$	1	
	$d^3 = 491018.04$		
	$d = 78.89\,mm$	2	
	$\theta = \frac{584\,Tl}{G d^4} = \frac{584\times6266.72\times10^3\times3.5\times10^3}{8\times10^4\times d^4}$	1	

	$d^4 = \dfrac{584 \times 6266.72 \times 10^3 \times 3.5 \times 10^3}{8 \times 10^4 \times 1}$ $d^4 = 160114696$ $d = 112.48mm$	2	
	Adopt diameter $d = 112.48mm$		7
7	Free B.M diagram has an ordinate of $\dfrac{Wl}{4}$ at the centre of each span. $A_1 = A_2 = \dfrac{1}{2} \times L \times \dfrac{Wl}{4} = \dfrac{Wl^2}{8}$ Applying theorem of three moment to span AB & BC. $M_A L_1 + 2M_B(L_1 + L_2) + M_C L_2 + \dfrac{6A_1\bar{x}_1}{L_1} + \dfrac{6A_2\bar{x}_2}{L_2} = 0$	1/2	
	Since the support A & C are S-S & $L_1 = L_2$ $M_A = M_C = 0$ $0 + 2M_B(2L) + 0 + \dfrac{6}{L} \times \dfrac{WL^2}{8} \times \dfrac{L}{2} + \dfrac{6}{L} \times \dfrac{WL^2}{8} \times \dfrac{L}{2} = 0$	1/2	
	$4M_B.L = -\dfrac{3}{4}\dfrac{WL^3}{L}$ $M_B = -\dfrac{3}{16}WL$ (Hogging moment)	1/2	

	Max BM at mid span		
	$= M_D = M_E = \frac{WL}{4} - \frac{1}{2}\left(\frac{3}{16} WL\right)$		
	$= \frac{+5}{32} WL$ (sagging)	$\frac{1}{2}$	
	Max negative BM @ support		
	$B = M_B = \frac{-3}{16} WL$	$\frac{1}{2}$	
	To obtain reaction taking moments about B for span BC we have		
	$R_C \times L - \frac{WL}{2} + \frac{3}{16} WL = 0$		
	$R_C = \frac{5}{16} W$	$\frac{1}{2}$	
	Taking moments about B for span AB we get		
	$R_A \times L = \frac{WL}{2} + \frac{3}{16} WL = 0$		
	$R_A = \frac{5}{16} W$		
	$(R_A + R_B + R_C) = W + W = 2W$		
	$R_B = 2W - (R_A + R_C)$		
	$= 2W - \frac{15}{16} W - \frac{15}{16} W$		
	$R_B = \frac{11}{8} W$		
	$R_A = R_C = \frac{\cancel{1}5}{16} W, \quad R_B = \frac{11}{8} W$	2	



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7

OR

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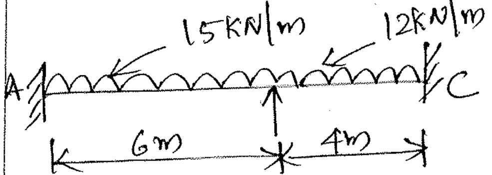


For AB

$$M_{FAB} = \frac{-wl^2}{12}$$

$$M_{FAB} = \frac{-15 \times 6^2}{12} = -45\ kNm$$

$$M_{FBA} = \frac{+wl^2}{12}$$

$$M_{FBA} = \frac{15 \times 6^2}{12} = 45\ kNm$$

For BC

$$M_{FBC} = \frac{-wl^2}{12} = \frac{-12 \times 4^2}{12} = -16.\ kN/m$$

$$M_{FCB} = \frac{+wl^2}{12} = \frac{-10 \times 3^2}{12} = +16.0\ kN/m$$

Relative stiffness of members

$K_{AB} = \frac{1}{6} \quad K_{BC} = \frac{3}{4} \cdot \frac{1}{3} = \frac{1}{4}$ (B hinged) 1

$\Sigma K = K_{AB} + K_{BC} = \frac{1}{6} + \frac{1}{4} = \frac{10}{24} I$

Calculation of distribution factors at joint 'B'

$\mu_{BA} = \frac{K_{AB}}{\Sigma K} = \frac{I/6}{10I/24}$

$= \frac{1}{6} \times \frac{24}{10I} = \underline{\underline{0.40}}$ 2

$\mu_{BC} = \frac{K_{BC}}{\Sigma K} = \frac{I/4}{10I/24} = \underline{\underline{0.6}}$

Joint	A	B	B	C
Distribution Factor		0.4	0.60	
Fixed end moment (initial)	-45	+45	-16	+16
				-16
Release C Carry over			-8.0	
Balance (Distribute)	-45	+45	-24	0
		-8.4	-12.6	
Carry over	-4.2			
	-49.2	+36.6	-36.6	0

2

Carry over moment to B $= \frac{-16.0}{2} = -8.0$ kNm

Unbalanced moment @ B

$= 45 - 24 = 21.0$

Distributed moment to AB

$= \mu_{BA} \times (-21.0)$

$= 0.4 \times -21 = -8.40$ kNm

Distributed moment to BC

$= \mu_{BC} \times (-21)$

$0.6 \times (-21) = -12.6$ kNm

Carry over moment to A $= \frac{-8.40}{2}$

$= -4.20$ kNm

$M_{AB} = -49.20$ kNm

$M_{BA} = 36.60$ kNm

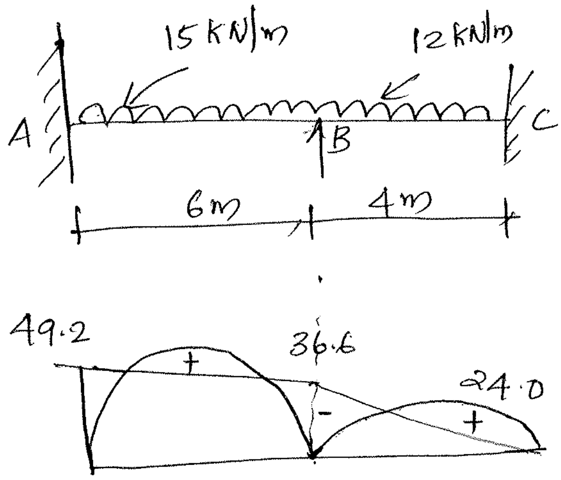
$M_{BC} = -36.60$ kNm

$M_{CB} = 0$

Max BM in span AB

$= \frac{Wl^2}{8} = \frac{15 \times 6^2}{8} = 67.5$ kNm

Max BM in span BC $= \frac{Wl^2}{8} = \frac{12 \times 4^2}{8} = 24$ kNm

		2	
9.	$\frac{M}{I} = \frac{f}{y} = \frac{E}{R}$	1	
	$M = Wl = 2\times10^3 \times 2000 = 4\times10^6$ N mm	2	
	$I = \frac{bd^3}{12} = \frac{40\times60^3}{12} = 72,0000$ mm^4	2	
	$f = \frac{My}{I}$, $f = \frac{4\times10^6\times30}{72,0000} = 166.66$ N/mm^2	2	7
	OR		
10	$P_c = 150$ kN $L = 1.50$m, $E = 200$ kN/m^2		
	$d = 3/4\ D$		
	$l_e = 2L = 2\times1.5 = 3.0$m	1	
	$I = \frac{\pi}{64}(D^4-d^4) = \frac{\pi}{64}\left[(D^4-0.75d^4)\right]$	1	
	$= 0.03356\,D4$		

	$P_{cr} = \dfrac{\pi^2 EI}{Le^2}$	2	
	$15 = \dfrac{\pi^2 \times 210 \times 0.03356\, D^4}{(3000)^2}$		
	$D^4 = \dfrac{15\,(3000)^2}{\pi^2\,(210 \times 0.03356)}$	1	
	$D = \dfrac{3}{4} \times 37.3 = \underline{\underline{27.975\,mm}}$	1	
	External diameter of strut = 37.3 mm	1	7
	Internal diameter of strut = 28.0 mm		
11	$M_A = M_B = \dfrac{WL}{8}$	2	
	$= \dfrac{50 \times 6}{8} = 37.5\ kN\text{-}m$	1	
	$y_{max} = \dfrac{WL^3}{192\,EI}$	2	
	$= \dfrac{50000 \times 6000^3}{192 \times 2.1 \times 10^5 \times 78 \times 10^6}$	1	
	$= \underline{\underline{3.434\,mm}}$	1	7
	OR		

12	- Portal frame consists of a beam resting over columns	1	
	- It is a statically indeterminate structure	1	
	- The main objective of portal frames is to reduce bending moment in the beam, which allows the frame to act as one structural unit.	1	
	- The transfer of stresses from the beam to the column results in the rotational movement at the foundation, which can be overcomed by introduction of pin/hinge joint.	1	
	- Portal frames are designed for roof load and wind load.	1	
	- They are constructed using steel, R.C.C and laminated timber	1	
	- The connections b/n the columns & the rafters are designed to be moment-resistant, ie, they carry bending forces.	1	7