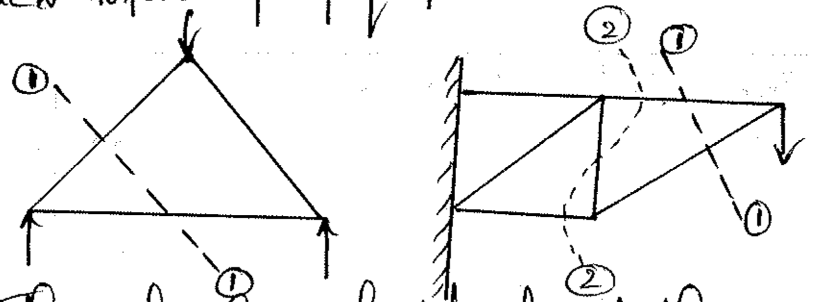
SCHEME OF VALUATION

Course Title : 4014-Theory of Structures II

Qn. No.	Scoring Indicator	Split-up Score	Sub Total	Total
	PART- A			
1)	Equivalent length of a column of a given length, given section, and given end conditions is defined as the length of the column of the same material, same section and having the same buckling load but having both of its ends hinged.	02	02	
2)	Check against (i) overturning (ii) sliding (iii) excessive compressive stress (iv) tensile stress	4x½	02	
3)	Beams which cannot be analysed fully using the conditions of static equillibrium are called statically indeterminate beams.	02	02	
4)	According to stiffness criteria the beam should be stiff enough to resist deflections. A beam is said to possess adequate stiffness when the deflection produced is within certain permissible limits expressed in relation to the span of the beam.	02	02	
5)	Distribution factor for a member at a joint is the ratio of the stiffness factor of the member to the total stiffness factor of all the members meeting at the joint.	02	02	10
	PART- B			
1)	Empirical equation proposed by Rankine $\frac{1}{P_R} = \frac{1}{P_c} + \frac{1}{P_e}$ P_R - Rankine's buckling load, P_c - crushing load $= f_c \cdot A$	01		

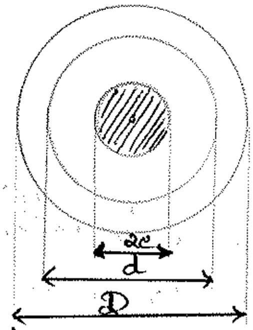
	P_E = Eulers buckling load			
	$= \dfrac{\pi^2 E I}{L_e^2}$			
	$P_R = \dfrac{P_e \times P_c}{P_e + P_c} = \dfrac{P_c}{1 + \frac{P_c}{P_E}}$	02		
	Substitute for P_E & P_c			
	$P_R = \dfrac{f_c \cdot A}{1 + \frac{f_c \cdot A \cdot L_e^2}{\pi^2 E I}}$			
	since $I = AK^2$			
	$P_R = \dfrac{f_c \cdot A}{1 + \frac{f_c}{\pi^2 E}\left(\frac{L_e}{K}\right)^2}$	02		
	$\dfrac{f_c}{\pi^2 E} = \alpha$, a constant			
	$P_R = \dfrac{f_c \cdot A}{1 + \alpha\left(\frac{L_e}{K}\right)^2}$	01	06	
	P_R - Rankines crippling load			
	A - Area of c.s. of column			
	L_e - Effective length of column			
	K - least radius of gyration.			
	α - Rankine's constant depends on E and f_c.			
2.	Method of sections : This method is suitable when the forces in a few members of the truss are required to be found out	02		
	Procedure			
	(i) Determine support reactions in case of simply supported trusses.			

(ii) a section line is passed through the members, in which forces are required to be found out.



(iii) The section line should not cut the members carrying more than three unknown forces.
(iv) a part of the structure on any side of the section line is then treated as a free body in equilibrium under the influence of external forces.
(v) unknown forces are evaluated by conditions of static equillibrium ie $\Sigma H = 0$, $\Sigma V = 0$, $\Sigma M = 0$ — 04 — 06

3)



02

$A = \frac{\pi}{4}(D^2 - d^2)$

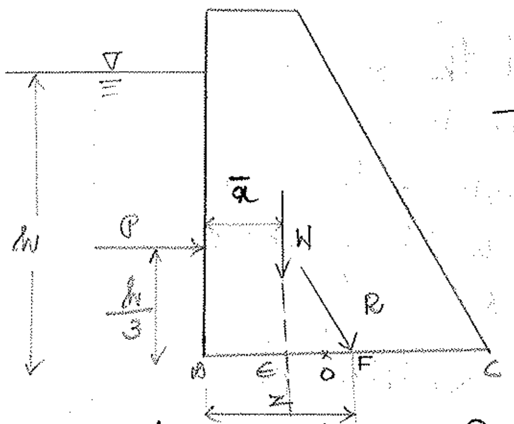
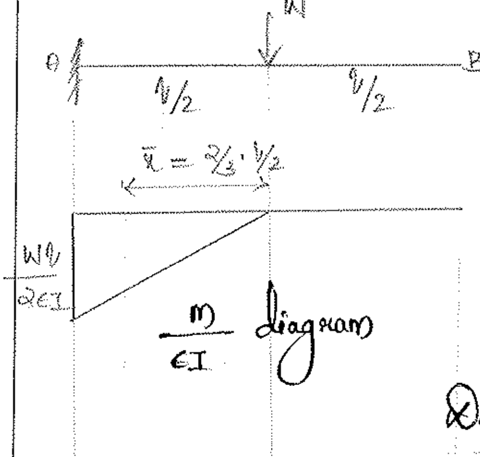
$I_{xx} = I_{yy} = \frac{\pi}{64}(D^4 - d^4)$

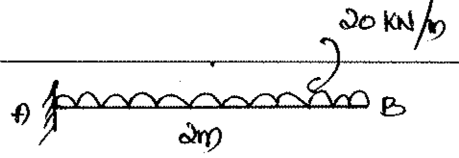
$Z_{xx} = Z_{yy} = \frac{\pi}{32}\left(\frac{D^4 - d^4}{D}\right)$ — 02

for no tension in the base

$$e = Z/A = \frac{\pi}{32}\left(\frac{(D^4 - d^4) \times 4}{(D^2 - d^2) \times \pi D}\right)$$

$$= \frac{1}{8}\left(\frac{D^4 - d^4}{D(D^2 - d^2)}\right) = \frac{D^2 + d^2}{8D}$$

	Core is a concentric circle with a diameter $2e = \frac{D^2+d^2}{4D}$	02	06	
4)	 Condition for no tension in the base is $e \le b/6$	03		
	Resultant should cut the base within b/6 from the centre of the base. $\therefore z = BO + OF = b/2 + e$ $= b/2 + b/6 = 2/3\, b$ i.e. $z = 2/3\, b$, the resultant should be within the middle third of the base for no tension to develop in the section.	03	06	
(5)	 Slope, θ_B = area of $\frac{M}{EI}$ diagram $= \frac{1}{2} \times \frac{l}{2} \cdot \frac{Wl}{2EI}$	02		
	$= \frac{Wl^2}{8EI}$	02		
	Deflection, y_B = area of $\frac{M}{EI}$ diagram $\times x = \frac{Wl^2}{8EI} \times \left(\frac{2}{3} \cdot \frac{l}{2} + \frac{l}{2}\right)$ $= 5Wl^3/48EI$	02	06	

6)				
	$\theta_B = \frac{wl^3}{6EI} = \frac{20\times 2^3}{6EI} = \frac{26.67}{EI}$	03		
	$y_B = \frac{wl^4}{8EI} = \frac{20\times 2^4}{8EI} = \frac{40}{EI}$	03	06	
7)	(i) Fixed end moments: The moments induced at the ends of the member due to the applied loads considering the member to be fixed at the both ends.	02		
	(ii) Unbalanced moments: The sum of fixed end moments meeting at a joint which is initially clamped is not zero. The algebraic sum of the moments meeting at a joint is called unbalanced moment which is responsible to rotate the unclamped joint.	02		
	(iii) Distributed moments: The resisting moments developed in the members are called distributed moments.	02	06	42
	PART - C			
III a)	Given, $A = 5490\ mm^2$, $L = 6m$ $I_{min} = I_{yy} = 5.108\times 10^6\ mm^4$ $E = 200\ KN/mm^2$ $f_c = 0.33\ KN/mm^2$, $\alpha = 1/7500$ Radius of gyration, $k = \sqrt{I_{yy}/A} = \sqrt{\frac{5.108\times 10^6}{5490}}$ $= 30.50\ mm$ Effective length, $L_e = L/\sqrt{2} = 6/\sqrt{2} = 4.24\ m$	02		

Crippling load by Rankine's formula,

$$P_R = \frac{f_c \cdot A}{1+\alpha\left(\frac{L_e}{k}\right)^2}$$

$$= \frac{0.33 \times 5490}{1+\frac{1}{7500}\times\left(\frac{4.24}{0.305}\right)^2} = 506.52\text{ KN}$$ (01)

Crippling load by Eulers formula =

$$P_E = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2 \times 210 \times 5.108 \times 10^6}{(4.24 \times 1000)^2}$$

$$= 588.89\text{ KN}$$ (01)

$P_E > P_R$.

$$P_E = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2 \times 210 \times 5.108 \times 10^6}{L_e^2} = \frac{1.059 \times 10^{10}}{L_e^2}$$ (01)

$$P_R = \frac{f_c \cdot A}{1+\alpha\left(\frac{L_e}{k}\right)^2} = \frac{0.33 \times 5490}{1+\frac{1}{7500}\times\frac{L_e^2}{30.5^2}}$$

$$= \frac{1811.7}{1+\frac{L_e^2}{6.97 \times 10^6}}$$ (01)

$P_E = P_R$

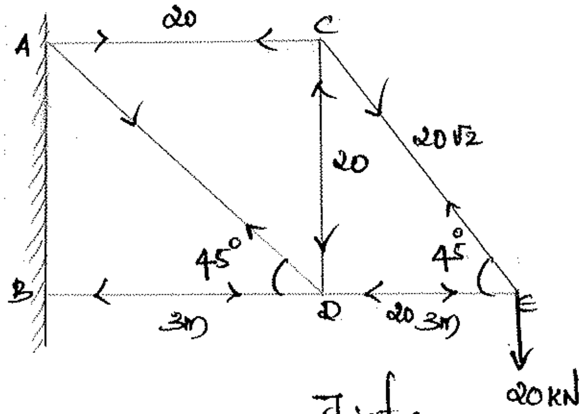
$$\frac{1.059 \times 10^{10}}{L_e^2} = \frac{1811.7}{1+\frac{L_e^2}{6.97 \times 10^6}}$$

$$1811.7 \cdot L_e^2 = (1.059 \times 10^{10}) + 1517.87\, L_e^2$$

On solving $L_e = 6\text{m}$, $L = \sqrt{2} \times 6 = 8.49\text{m}$.

$P_E = P_R$, when length of strut $= 8.49\text{m}$. (03) (09)

III b)



Joint E

$F_{EC} = 20\sqrt{2}$ (T)

$F_{ED} = 20$ (C)

Joint C

$F_{CD} = 20$ (C)

$F_{CA} = 20$ (T)

3x02 06

Joint D

$F_{DA} = 20\sqrt{2}$ (T)

$F_{DB} = 20$ (C)

Members	Forces (KN)
EC	$20\sqrt{2}$ (T)
ED	20 (C)
CD	20 (C)
CA	20 (T)
DA	$20\sqrt{2}$ (T)
DB	20 (C)

15

IV a)

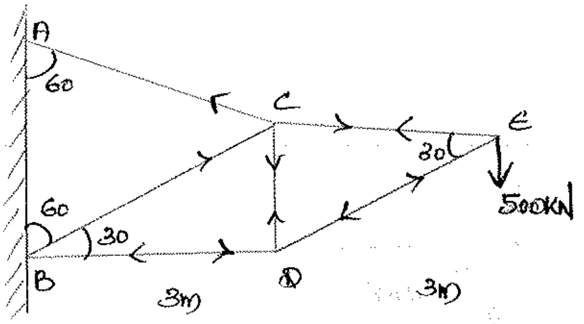
$$P_E = \frac{\pi^2 EI}{l_e^2}$$ 02

$$P_E \propto I$$ 01

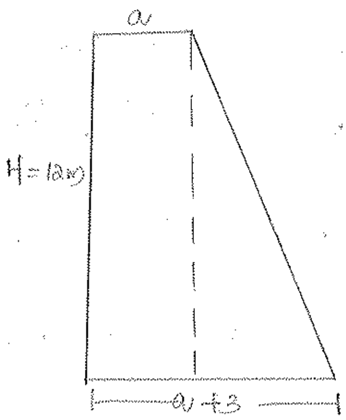
$$\frac{P_H}{P_S} = \frac{\frac{\pi}{64}\left(D^4 - \left(\frac{D}{2}\right)^4\right)}{\frac{\pi}{64}\left(D^4\right)}$$ 02

$$= 1 - \frac{1}{16}$$

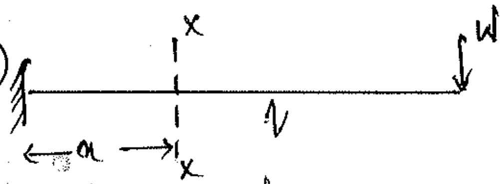
$$\frac{P_H}{P_S} = 0.9375$$ 01 06

IV b)	 **Joint E** $F_{ED} = 500/\sin 30 = 1000\text{ KN (C)}$ $F_{CE} = 1000 \times \cos 30 = 866.025\text{ KN (T)}$ **Joint D** $F_{DB} = 1000 \cos 60 = 500\text{ KN (C)}$ $F_{CD} = 1000 \sin 60 = 500\text{ KN (T)}$ **Joint C** $F_{CA} \sin 30 + F_{CB} \sin 30 = 500$ $F_{CA} + F_{CB} = 1000 \longrightarrow (1)$ $-F_{CA} \cos 30 + F_{CB} \cos 30 = -866.025$ $F_{CB} - F_{CA} = -1000 \longrightarrow (2)$ $F_{CB} = 0, \quad F_{CA} = 1000\text{ KN (T)}$	02 02 03		
	Member / Force (KN) ED: 1000 (C) EC: 866.025 (T) DC: 500 (T) BD: 866.025 (C) CB: 0 CA: 1000 (T)	02	09	15

V a)	$A = (200\times 200) - (120\times 120) = 25660\ mm^2$	01		
	$Z = I/y_{max} = \dfrac{200^4 - 120^4}{12\times(200/2)}$ $= 1.16\times 10^6\ mm^3$	02		
	$\left.\begin{matrix} f_{max} \\ f_{min} \end{matrix}\right\} = \dfrac{P}{A} \pm \dfrac{Pe}{Z}$ $60 = P\left(\dfrac{1}{25660} + \dfrac{50}{1.16\times 10^6}\right)$ $P = 731.042$ KN	03	06	
V b)	$H = 12m$ $b = a + 12/4 = a + 3m$	01		
	$P = \dfrac{\gamma H^2}{2} = \dfrac{10\times 12^2}{2} = 720$ KN	02		
	$W = \dfrac{1}{2}H(a+b)\times 22$ $= (2a+3)\times 132$	01		
	$\bar{x} = \dfrac{a^2 + ab + b^2}{3(a+b)} = \dfrac{a^2 + a(a+3) + (a+3)^2}{3(2a+3)}$	02		
	$z = \bar{x} + \dfrac{P}{W}\cdot\dfrac{H}{3}$	01		
	For no tension ; $z = 2/3\, b = 2/3\,(a+3)$ $\dfrac{a^2 + a(a+3) + (a+3)^2}{3(2a+3)} + \dfrac{720\times 4}{(2a+3)\times 132} = 2/3\,(a+3)$ On solving $a = 4.258m$, $b = 7.258m$.	02	09	15



VI a)	Given $P = 2000\ kN$			
	$A = \frac{\pi}{4} \times (30^2 - 20^2) = 392.69\ cm^2$	01		
	$Z = \frac{\pi}{64} \frac{(30^4 - 20^4)}{15} = 2127.12\ cm^3$	01		
	For tension $= 0$			
	$\frac{P}{A} - \frac{Pe}{Z} = 0$			
	$\frac{P}{A} = \frac{Pe}{Z}$	02		
	$\frac{2000}{392.69} = \frac{2000 \times e}{2127.12}$			
	$e = 5.42\ cm.$	02	06	
VI b)	Given			
	$a = 2m \quad b = 6m \quad H = 12m \quad \gamma = 18\ kN/m^3$			
	$\gamma_m = 24\ kN/m^3, \ \phi = 30^\circ, \ K_a = \frac{1-\sin\phi}{1+\sin\phi} = \frac{1}{3}$	01		
	$P = \frac{\gamma H^2}{2} \times \frac{1}{3} = \frac{18 \times 12^2}{2} \times \frac{1}{3} = 432\ kN$	01		
	$W = \frac{1}{2} H (a+b) \gamma_m = \frac{1}{2} \times 12 \times (2+6) \times 24$			
	$= 1152\ kN$	01		
	$\bar{x} = \frac{a^2 + ab + b^2}{3(a+b)} = 2.167\ m.$	01		
	$z = \bar{x} + \frac{P}{W} \cdot \frac{H}{3}$			
	$= 3.667\ m$	01		
	$e = z - \frac{b}{2} = 0.667\ m.$	01		

	$\left.\begin{matrix} f_{max} \\ f_{min} \end{matrix}\right\} = \frac{W}{b}\left(1 \pm \frac{6e}{b}\right)$			
	$= \frac{1152}{6}\left(1 \pm \frac{6 \times 0.667}{6}\right)$			
	$= 320.06\ KN/m^2$			
	$63.936\ KN/m^2$	02	09	15
VII(a)		01		
	Given: $l = 2m$, $l' = 1.2m$, $w = 2\ KN/m$, $W = 1\ KN$, $E = 11\ KN/mm^2 = 11 \times 10^6\ KN/m^2$, $I = 66 \times 10^6\ mm^4 = 66 \times 10^{-6}\ m^4$			
	$y_B = \frac{Wl^3}{3EI} + \frac{wl'^4}{8EI} + \frac{wl'^3}{6EI}(2-1.2)$	02		
	$= \frac{1}{11 \times 10^6 \times 66 \times 10^{-6}}\left(\frac{1 \times 2^3}{3} + \frac{2 \times 1.2^4}{8} + \frac{2 \times 1.2^3 \times 0.8}{6}\right)$	02		
	$= 5.02 \times 10^{-3}\ m$			
	$= 5.02\ mm.$	02	06	
VII(b)		01		
	Bending moment @ secn $x-x = -W(l-x)$			
	$EI\frac{d^2y}{dx^2} = M$			
	$EI.\frac{d^2y}{dx^2} = -W(l-x)$	01		
	Integrating			
	$EI.\frac{dy}{dx} = -W\left(lx - \frac{x^2}{2}\right) + C_1$			
	At $x = 0$, $\theta = \frac{dy}{dx} = 0$			
	$C_1 = 0$	01		

Thus $EI.\frac{dy}{dx} = -W\left(lx - \frac{x^2}{2}\right)$

Slope, $\frac{dy}{dx} = \frac{-W}{2EI}\left(2lx - x^2\right)$

$EI.\frac{dy}{dx} = -W\left(lx - \frac{x^2}{2}\right)$ — 01

Integrating

$EI.y = -W\left(\frac{lx^2}{2} - \frac{x^3}{6}\right) + C_2$

@ $x=0$ $y=0$

$C_2 = 0$

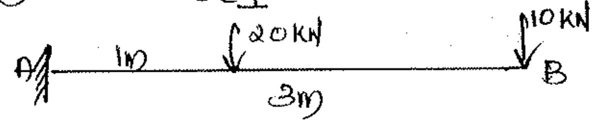
Deflection, $y = \frac{-W}{6EI}\left(3lx^2 - x^3\right)$ — 01

Slope and deflection @ free end ie $x = l$

$\frac{dy}{dx} = \frac{-W}{2EI}\left(2l^2 - l^2\right) = \frac{-Wl^2}{2EI}$ — 02

$y = \frac{-W}{6EI}\left(3l^3 - l^3\right) = \frac{-Wl^3}{3EI}$ — 02 — 09 — 15

VIII a)



— 01

$E = 200\,KN/mm^2 = 200 \times 10^6\,KN/m^2$, $W = 10KN$

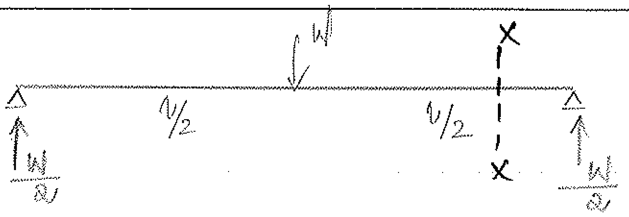
$I = 150 \times 10^6\,mm^4 = 150 \times 10^{-6}\,m^4$, $W_1 = 20KN$ — 01

$l = 3m$ $l_1 = 1m$

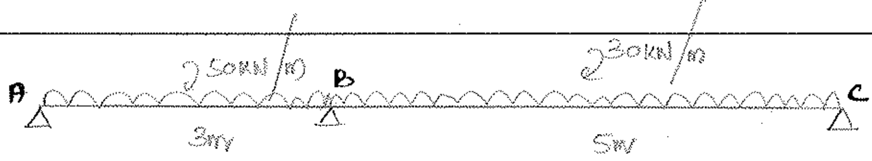
$y_B = \frac{Wl^3}{3EI} + \frac{W_1 l_1^3}{3EI} + \frac{W_1 l_1^2}{2EI} \times 2m$ — 02

$= \frac{1}{200\times10^6 \times 150\times10^{-6}}\left(\frac{10\times3^3}{3} + \frac{20\times1^3}{3} + \frac{20\times1^2}{2}\times2\right)$

$= 3.88\times10^{-3}\,m = 3.88\,mm$ — 02 — 06

VIII b)		01		
	$M_x = \frac{W}{2}x - W\left(x - \frac{l}{2}\right)$			
	$EI.\frac{d^2y}{dx^2} = \frac{Wx}{2} - W\left(x - \frac{l}{2}\right)$	01		
	Integrating twice			
	$EI.\frac{dy}{dx} = \frac{Wx^2}{4} - \frac{W}{2}\left(x - \frac{l}{2}\right)^2 + C_1$			
	$EI.y = \frac{Wx^3}{12} - \frac{W}{6}\left(x - \frac{l}{2}\right)^3 + C_1x + C_2$			
	@ $x = 0$, $y = 0 \implies C_2 = 0$	01		
	@ $x = l$, $y = 0$			
	$0 = \frac{Wl^3}{12} - \frac{W}{6} \times \frac{l^3}{8} + C_1 . l$			
	Solving, $C_1 = \frac{-Wl^2}{16}$	01		
	$\frac{dy}{dx} = \frac{1}{EI}\left(\frac{Wx^2}{4} - \frac{W}{2}\left(x - \frac{l}{2}\right)^2 - \frac{Wl^2}{16}\right)$			
	$y = \frac{1}{EI}\left(\frac{Wx^3}{12} - \frac{W}{6}\left(x - \frac{l}{2}\right)^3 - \frac{Wl^2x}{16}\right)$	01		
	Maximum slope @ $x = 0$			
	$\theta_{max} = \frac{-Wl^2}{16EI}$	02		
	y_{max} @ $x = l/2$			
	$y_{max} = 1/EI\left(\frac{W}{12} \times \frac{l^3}{8} - \frac{Wl^2}{16} . \frac{l}{2}\right)$			
	$= -Wl^3/48EI$	02	09	15

IX a)



$L_1 = 3m \qquad L_2 = 5m$

For AB:

Max free BM $= \dfrac{50 \times 3^2}{8} = 56.25$ KN-m

$A_1 = \frac{2}{3} \times 56.25 \times 3 = 112.5 \text{ KN m}^2$

$\bar{x}_1 = 3/2 = 1.5m$

For BC

Max free BM $= \dfrac{30 \times 5^2}{8} = 93.75$ KN-m

$A_2 = \frac{2}{3} \times 93.75 \times 5 = 312.5 \text{ KN-m}^2$

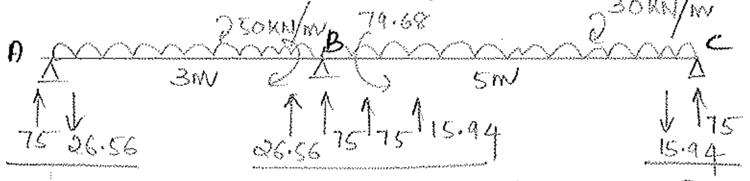
$\bar{x}_2 = 5/2 = 2.5m.$ 02

Apply three moment theorem.

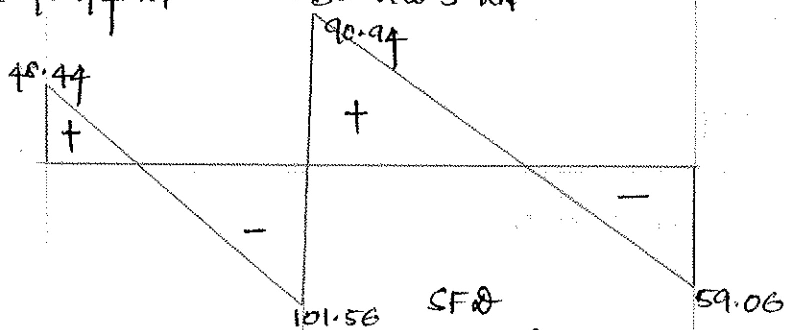
$$M_A . L_1 + 2M_B(L_1 + L_2) + M_C . L_2 + \frac{6A_1\bar{x}_1}{L_1} + \frac{6A_2\bar{x}_2}{L_2} = 0$$

$$2M_B \times 8 = -\left(\frac{6 \times 56.25 \times 1.5}{3} + \frac{6 \times 312.5 \times 2.5}{5}\right)$$

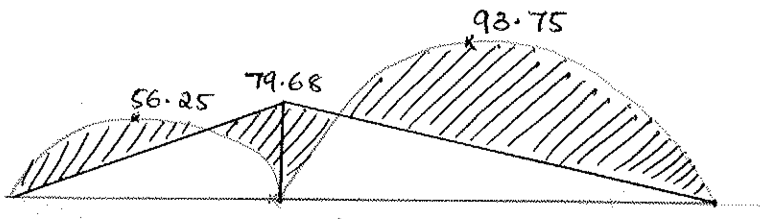
$$M_B = -79.68 \text{ KN-m}.$$ 02



$R_A = 48.44$ KN $\qquad R_B = 192.5$ KN $\qquad R_C = 59.06$ KN 01

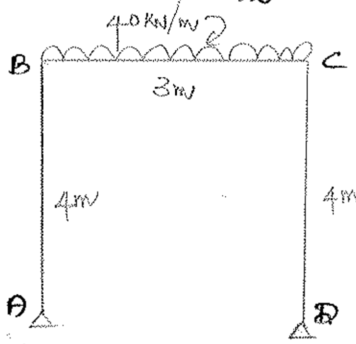


01



BMD 01 07

IX b)



01

$$M_{FBC} = \frac{-wl^2}{12} = \frac{-40\times 3^2}{12} = -30\ \text{kN-m}$$ 01

$M_{FCB} = 30\ \text{kN-m}.$ 01

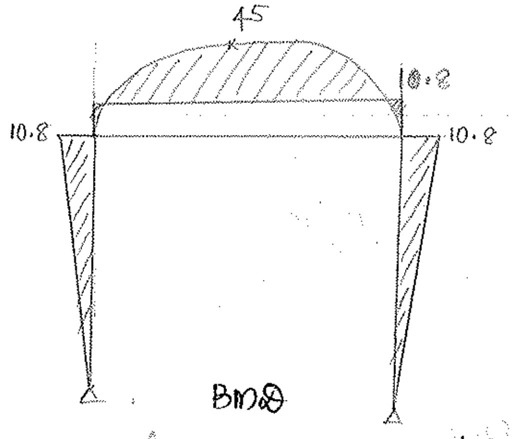
Distribution factor

Joint	Member	k	Σk	DF
B	BA	$\frac{3}{4}\frac{I}{4}$	0.52 I	0.36
	BC	$I/3$		0.64
C	CD	$\frac{3}{4}\frac{I}{4}$	0.52 I	0.36
	CB	$I/3$		0.64

02

A	B	B	C	C	D
	0.36	0.64	0.64	0.36	
		-30	30		
	10.8	19.2	-19.2	-10.8	
0	10.8	-10.8	10.8	-10.8	0

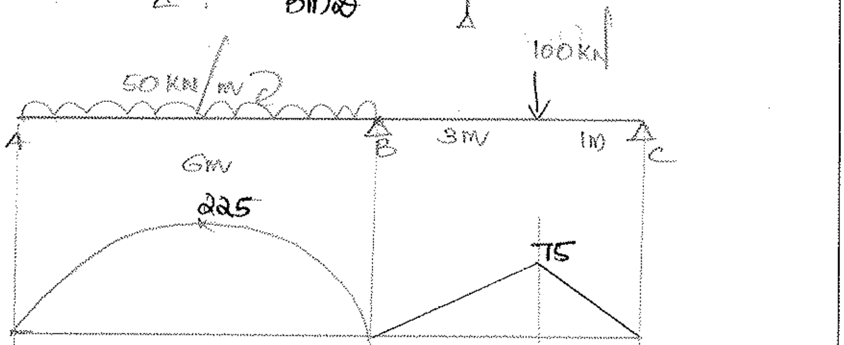
02



BMD

02 | 08 | 15

X a)



$A_1 = \frac{2}{3} \times 225 \times 6$

$= 900\ kN\text{-}m^2$

$\bar{x}_1 = 6/2 = 3m$

$A_2 = \frac{1}{2} \times 4 \times 75 = 150\ kN\text{-}m^2$

$\bar{x}_2 = \frac{4+1}{3} = 5/3\ m$

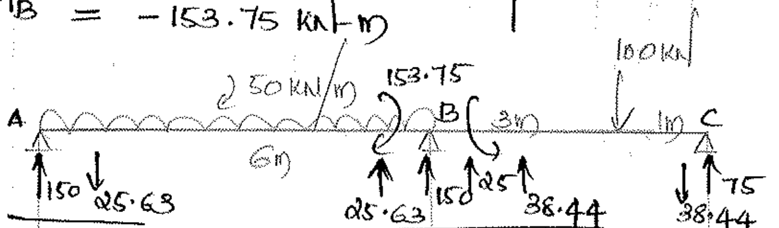
02

$$M_A \cdot L_1 + 2M_B(L_1+L_2) + M_C \cdot L_2 + \frac{6A_1\bar{x}_1}{L_1} + \frac{6A_2\bar{x}_2}{L_2} = 0$$

$$2M_B \times 10 + \frac{6 \times 900 \times 3}{6} + \frac{6 \times 150 \times 5}{3 \times 4} = 0$$

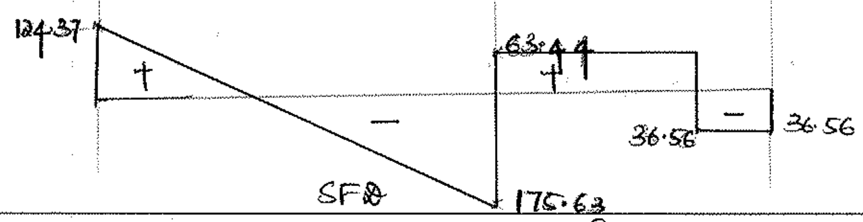
$$M_B = -153.75\ kN\text{-}m$$

02



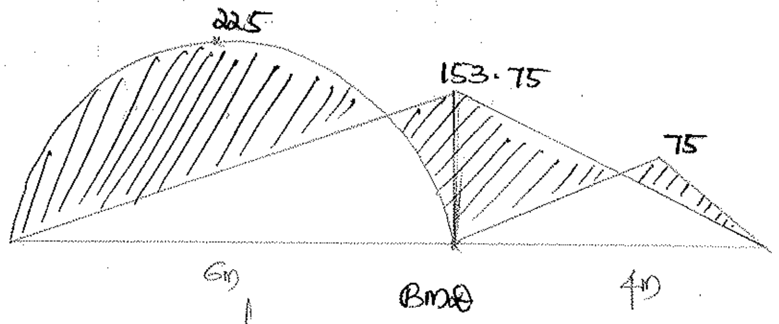
$R_A = 124.37$ $\quad R_B = 239.07$ $\quad R_C = 36.56$

01



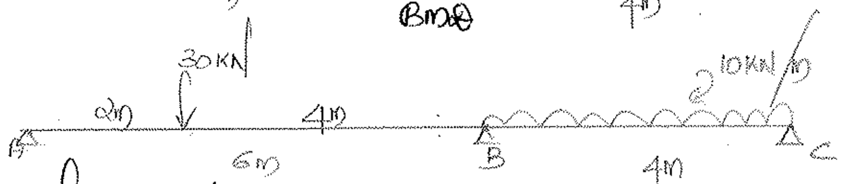
SFD

01



01 | 07

IX b)



01

Fixed End Moments

$M_{FAB} = \frac{-Wab^2}{l^2} = \frac{-30 \times 2 \times 4^2}{6^2} = -26.67$ KN-m

$M_{FBA} = \frac{Wa^2b}{l^2} = \frac{30 \times 2^2 \times 4}{6^2} = 13.33$ KN-m

$M_{FBC} = \frac{-wl^2}{12} = \frac{10 \times 4^2}{12} = -13.33$ KN-m

$M_{FCB} = 13.33$ KN-m

02

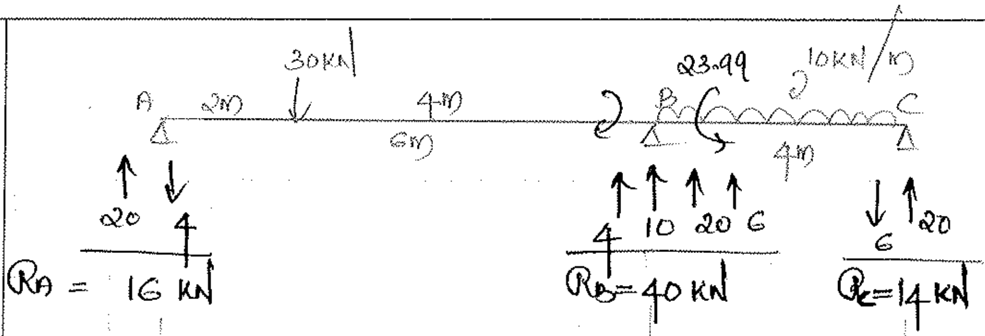
Distribution Factor

Joint	Members	k	Σk	DF
B	BA	$\frac{3}{4}\frac{I}{6}$	$\frac{5}{16}I$	0.4
	BC	$\frac{3}{4}\frac{I}{4}$		0.6

01

A	B		C
	0.4	0.6	
-26.67	13.33	-13.33	13.33
			-13.33
26.67	13.33	-6.66	
0	26.66	-19.99	0
	-2.67	-4.002	
	23.99	-23.99	

02



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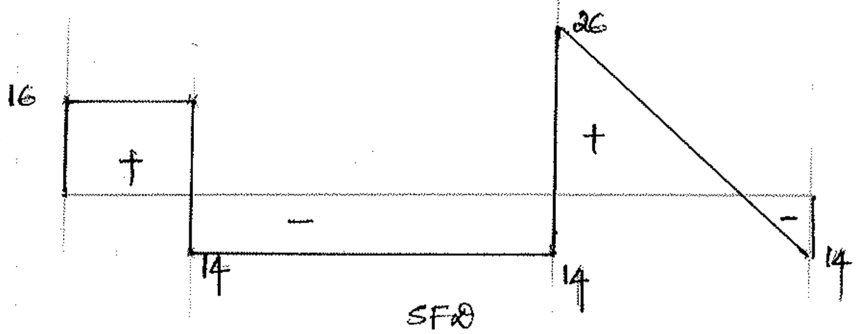
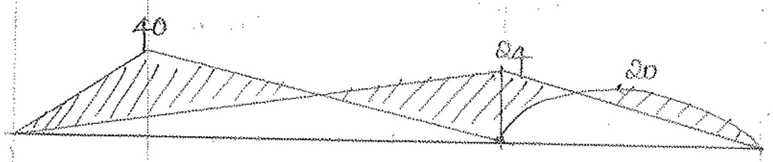


SFD

01



BMD

01 08 15

———— x ———— x ————