| | SCEME OF VALUATION | | | |
|---|---|---|---|---|
| (Scoring indicators) | | | | |
| | | | | |
| Revision : **2015** | | Course Code : **3013** | | |
| Course Title: **THEORY OF STRUCTURES I** | | | | |
| | | | | |
| | | | | |
| Question No. | Scoring Indicator | Split up score | Sub Total | Total |
| | | | | |
| | PART A | | | |
| I(1) | Moment of a force about a point is defined as the product of that force (F) and perpendicular distance between the point and force or line of action of force (d).<br>ie, M = F x d | 2 | 2 | 10 |
| I(2) | Centroid may be defined as the point at which total area of the plane figure is assumed to be concentrated | 2 | 2 | |
| I(3) | Hooke's law states that when a material is loaded within elastic limit, the stress is proportional to the strain produced by the stress. | 2 | 2 | |
| I(4) | $T/J = q/R = G\theta/l$.<br><br>Where T= Maximum twisting moment (Torque),<br>J = Polar moment of inertia,<br>q = shear stress,<br>R = Radius of the shaft,<br>G = modulus of rigidity,<br>θ = Angle of twist,<br>l = Length of shaft | 2 | 2 | |
| I(5) | $M/I = f/y = E/R$,<br><br>Where M = Bending moment,<br>I = Moment of Inertia,<br>f = Bending stress,<br>y = Distance of fiber from neutral axis,<br>E = Modulus of elasticity,<br>R = Radius of curvature. | 2 | 2 | |

P2

PART B.

| | | | | |
|---|---|---|---|---|
| II | 1 | i. Point load<br>ii Uniformly distributed load<br>iii Uniformly varying load<br>iv Rolling load. | Any 3<br>3. | |
| | | (i) Point load – Load acting at a definite point on a beam is known as a point load<br>(ii) udl – A load which is spread over a beam in such a manner that each unit length is loaded to the same intencity is known as uniformly distributed load.<br>(iii) uvl – A load which is spread over a beam in such a manner that the intencity of loading varies uniformly on each unit length is known as uniformly varying load.<br>(iv). Rolling loads – Rolling loads are those loads which roll over the given structural element from one end to the another. | Any 3<br>3 | 6 |
| II | 2 | The moment of inertia of a lamina ~~about a~~ ~~line~~ about a line parallel to its centroidal line is equal to its moment of inertia about its own centroid plus area of lamina × (distance)$^2$, where the distance is the distance between centroid and that line. | 2 | |
| | | Given that $D = 50$ mm<br>$d = 30$ mm | | |
| | | $I_{xx} = \frac{\pi}{64}(D^4 - d^4)$ | 1 | |
| | | $I_{yy} = \frac{\pi}{64}(D^4 - d^4)$ | 1 | |
| | | Polar moment of Inertia $= I_{\cancel{zz}} = I_{xx} + I_{yy}$ | 1 | 6 |
| | | $I_{zz} = \frac{\pi}{64}(D^4 - d^4) + \frac{\pi}{64}(D^4 - d^4)$<br>$= \frac{2\pi}{64}(D^4 - d^4)$<br>$= \frac{\pi}{32}(D^4 - d^4)$<br>$\therefore I_{zz} = \frac{\pi}{32}(D^4 - d^4)$<br>$= \frac{\pi}{32}(50^4 - 30^4)$ | | |
| | | $I_{zz} = 53.407 \times 10^4 \text{ mm}^4$ | 1 | |

II (3) Given that $P = 10 kN = 10\times10^3 N$, $E = 2\times10^5 N/mm^2$

Area of first part $A_1 = \frac{\pi}{4}\times 20^2 = 314.00 mm^2$ 1

Area of second part $A_2 = \frac{\pi}{4}\times 10^2 = 78.54 mm^2$ 1

Stress in the first part $= f_1 = \frac{P}{A_1}$

$$= \frac{10\times10^3}{314.00}$$ 1

$$= 31.85 N/mm^2$$

Stress in the second part $= f_2 = \frac{P}{A_2}$ 1

$$= \frac{10\times10^3}{78.54}$$ 6

$$= 127.32 N/mm^2$$

Given that $l_1 = 250 mm$ & $l_2 = 200 mm$.

Total extension of the bar, $\delta l = \frac{f_1}{E} l_1 + \frac{f_2}{E} l_2$.

$$= \frac{1}{E}(f_1 l_1 + f_2 l_2)$$ 1

$$= \frac{1}{2\times10^5}(31.85\times250 + 127.32\times200)$$

$$\delta l = \underline{\underline{0.167 mm}}$$ 1

II (4)

Given that $P = 10 kN = 10\times10^3 N$.

$l = 4m = 4000 mm$.

$d = 20mm$

$A = \frac{\pi d^2}{4} = \frac{\pi\times 20^2}{4} = 314.00 mm^2$ 1

$E = 200 kN/mm^2 = 200\times10^3 N/mm^2$

Since the load is applied gradually

$$f = \frac{P}{A}$$ 1

$$= \frac{10\times10^3}{314}$$

$$f = 31.847 N/mm^2$$ 1

Volume, $V = A\times l$ 6

$$= 314\times4000$$ 1

$$= 1.256\times10^6 mm^3$$

Strain energy stored $= \frac{f^2}{2E}\times V$ 1

$$= \frac{31.847^2}{2\times2\times10^5}\times 1.256\times10^6$$

$$= 3184.69 Nmm$$ 1

$$= 3.184 Nm$$

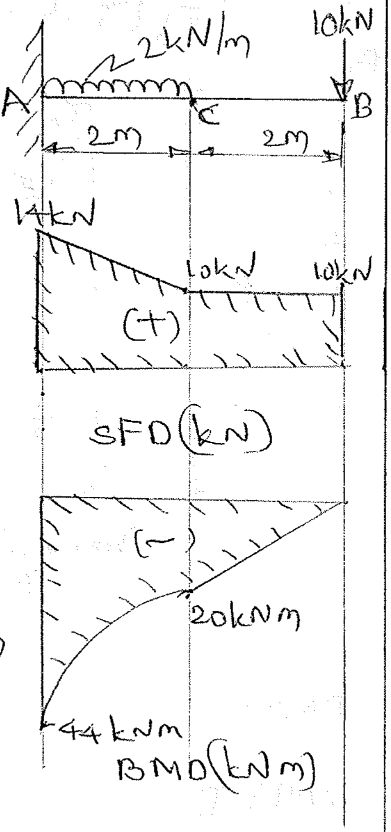
II (5) Calculation of SF
Let Right upward forces as positive.
(PUL & RUN)

SF at B = +10 kN

SF at C = +10 kN

SF at A = +10 + 2×2
= +14 kN.

Calculation of BM
Let Right anticlockwise as positive.
BM at B = 0
BM at C = −10×2 = −20 kNm
BM at A = −10×4 + (−2×2)×1
= −40 + −4
= −44 kNm




3

3

6

II (6)
1. The material of the shaft is homogeneous and isotropic.
2. Cross section of the shaft which are plane before twist remain plane after twist.
3. Twist along the length of the shaft is uniform throughout.
4. Circular sections remain circular after loading
5. Hooke's law is applicable with shear force being proportional to the shearing strain.
6. Shafts are subjected to twisting couples in planes that are perpendicular to the axis of the shaft.

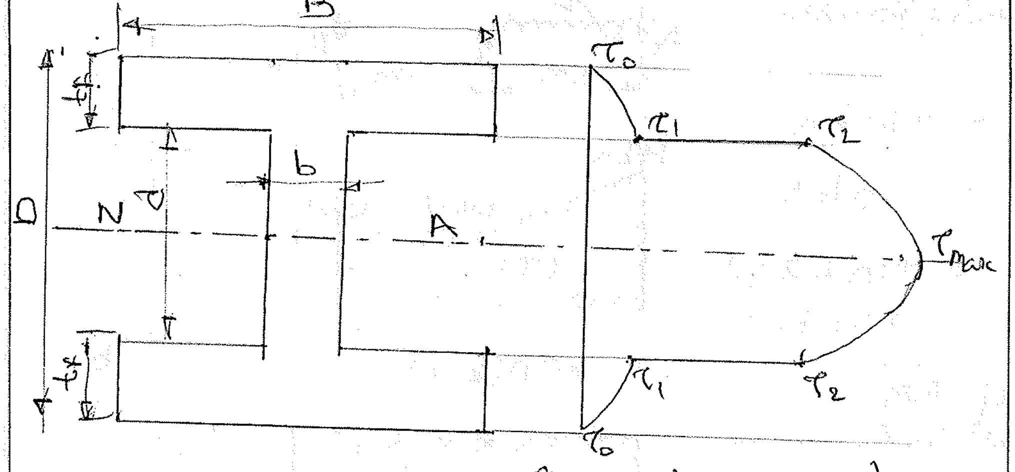
Any 6

6 | 6

II (7) Shear stress at the upper edges of the flange, $\tau_0 = 0$
Shear stress at the lower edge of the flange, $\tau_1 = \frac{F}{8I}(D^2 - d^2)$
Shear stress at the junction of top of the web and bottom flange $= \frac{B}{b}\frac{F}{8I}(D^2 - d^2)$

Shear stress at the centre of web = Maximum 3

Shear stress $\tau_{max} = \frac{F}{Ib}\left[\frac{B}{8}(D^2-d^2) + \frac{bd^2}{8}\right]$

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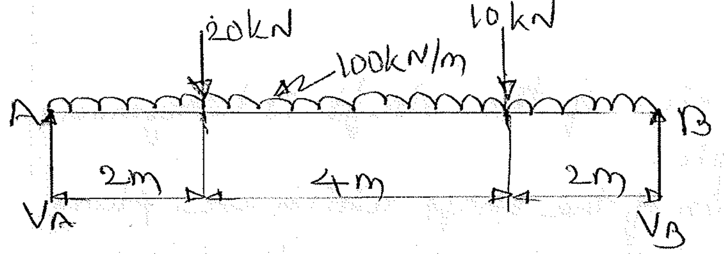
Shear stress Distribution Diagram.

PART C

UNIT I

III (a)




1

Let $V_A$ & $V_B$ be the support reactions at A and B respectively.

Applying the conditions of static equilibrium,

$\Sigma V = 0$

ie, $V_A + V_B = 20 + 10 + 100 \times 8$ 1

$V_A + V_B = 830\text{ kN} \rightarrow (1)$ 1

$\Sigma M_A = 0$

$V_B \times 8 + -10 \times 6 + -20 \times 2 + (-100 \times 8) \times 4 = 0$ 1 6

$8V_B - 60 - 40 - 3200 = 0$

$8V_B = 3300$

$V_B = \frac{3300}{8}$

$V_B = 412.5\text{ kN}$ 1

Put $V_B = 412.5$ kN in equation (1)

$V_A + 412.5 = 830$

$V_A = 830 - 412.5$ 1

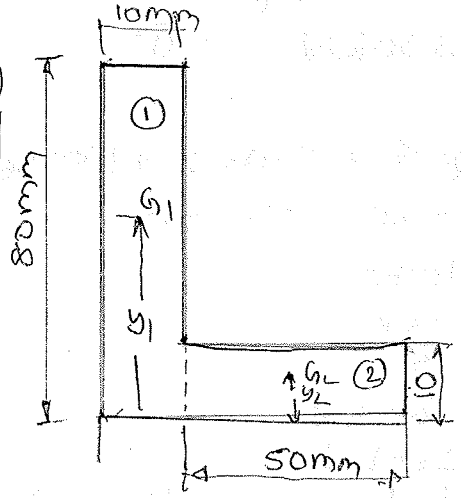
$V_A = 417.5\text{ kN}$

$\boxed{V_A = 417.5\text{ kN} \\ V_B = 412.5\text{ kN}}$

III (b)

$\bar{y}$

$a_1 = 10 \times 80 = 800\,mm^2$
$y_1 = \frac{80}{2} = 40\,mm$
$a_2 = 50 \times 10 = 500\,mm^2$
$y_2 = \frac{10}{2} = 5\,mm.$ 1




$\therefore \bar{y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2}$ 1

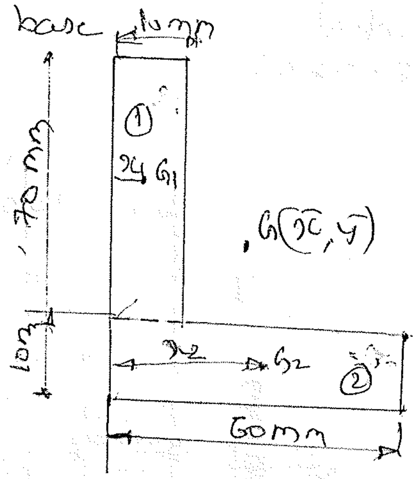
$= \frac{800 \times 40 + 500 \times 5}{800 + 500}$

$= 26.54\,mm$

$\bar{y} = 26.54\,mm$ from base 1

$\bar{x}$

$a_1 = 10 \times 70 = 700\,mm^2$
$x_1 = \frac{10}{2} = 5\,mm$ 1
$a_2 = 60 \times 10 = 600\,mm^2$
$x_2 = \frac{60}{2} = 30\,mm.$ 1




$\therefore \bar{x} = \frac{a_1 x_1 + a_2 x_2}{a_1 + a_2}$ 1 9

$= \frac{700 \times 5 + 600 \times 30}{700 + 600}$

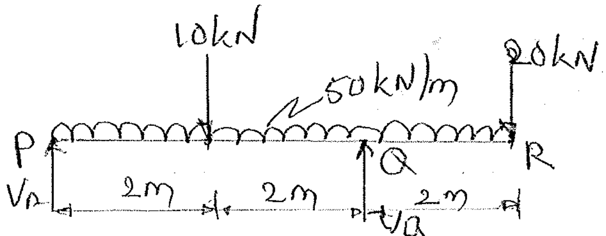
$= 16.54\,mm$ 1

$\bar{x} = 16.54\,mm$ from left end.

$\therefore$ Centroid $G(\bar{x}, \bar{y}) = (16.54\,mm, 26.54\,mm)$ 1 15

16.54mm from left end, 26.54mm from base

IV (a)




1

Let ~~P~~ $V_P$ & $V_Q$ be the support reactions at P & Q respectively.
Applying the conditions of static equilibrium,

$\Sigma V = 0$

ie, $V_P + V_Q = 10 + 20 + (50 \times 6)$ | 1

ie, $V_P + V_Q = 330\,kN \rightarrow (1)$ | 1

$\Sigma M_P = 0$

$-20 \times 6 + V_Q \times 4 + -10 \times 2 + (50 \times 6) \times 3 = 0$ | 1

$4V_Q - 120 - 20 - 900 = 0$

$4V_Q = 1040$

$V_Q = \frac{1040}{4}$ | 6

$= 260\,kN$ | 1

$V_Q = 260\,kN$

Put $V_Q = 260\,kN$ in equation (1)

$V_P + 260 = 330$

$\therefore V_P = 330 - 260$ | 1

$V_P = 70\,kN$

$\therefore V_P = 70\,kN$

$V_Q = 260\,kN$.

IV (b)

Given that

$B = 100mm$

$D = 200mm$

$t_f = 15mm$

$t_w = 10mm$

$d = 200 - 2t_f$

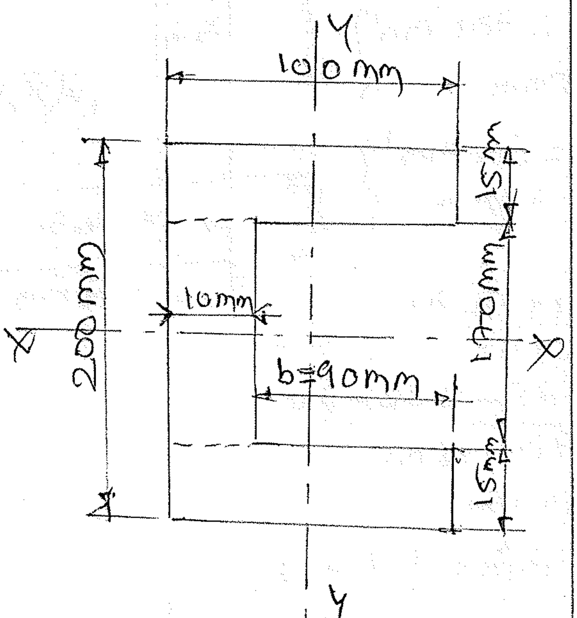
$= 200 - 15 - 15$

$= 170mm$ | 1

$b = B - t_w$

$= 100 - 10$ | 1

$= 90\,mm$




Section is symmetrical about XX

$I_{xx} = \frac{1}{12}BD^3 - \frac{1}{12}bd^3 = \frac{1}{12}[BD^3 - bd^3]$ | 1

$= \frac{1}{12}[100 \times 200^3 - 90 \times 170^3]$

$= 29.819 \times 10^6\,mm^4$

$\underline{I_{xx} = 29.819 \times 10^6\,mm^4}$ | 1

$\underline{I_{yy}}$

$\bar{x} = ?$

$a_1 = 100 \times 15 = 1500\,mm^2$

$x_1 = \frac{100}{2} = 50\,mm.$

$a_2 = 10\times170 = 1700\,mm^2$
$x_2 = \frac{10}{2} = 5\,mm$ | 1

$a_3 = 100\times15 = 1500\,mm^2$
$x_3 = \frac{100}{2} = 50\,mm$

$$\bar{x} = \frac{a_1x_1+a_2x_2+a_3x_3}{a_1+a_2+a_3}$$

$$= \frac{1500\times50+1700\times5+1500\times50}{1500+1700+1500}$$ | 1

$\bar{x} = \underline{\underline{33.72\,mm}}$ from left end

$I_{xx} = I_{xx1}+A_1h_1^2+I_{xx2}+A_2h_2^2+I_{xx3}+A_3h_3^2$ | 1

$h_1 = \bar{x}-x_1 = 33.72-50 = -16.28\,mm$
$h_2 = \bar{x}-x_2 = 33.72-5 = 28.72\,mm$ | 1
$h_3 = \bar{x}-x_3 = 33.72-50 = -16.28\,mm$

$$I_{xx} = \frac{d_1b_1^3}{12}+A_1h_1^2+\frac{d_2b_2^3}{12}+A_2h_2^2+\frac{d_3b_3^3}{12}+A_3h_3^2$$

$$= \frac{15\times100^3}{12}+1500\times(-16.28)^2+\frac{170\times10^3}{12}+1700\times28.72^2$$
$$+\frac{15\times100^3}{12}+1500\times(-16.28)^2$$ | 9

$= \underline{\underline{4.712\times10^6\,mm^4}}$ | 1

$I_{xx} = 29.819\times10^6\,mm^4$
$I_{yy} = 4.712\times10^6\,mm^4$ | 15

UNIT II

V (a)

1. Strength – Ability of a material to resist breakdown or yielding under the action of forces.
2. Stiffness – Ability of a material to resist deformation when it is stressed
3. Elasticity – Property of a material to regain its original shape after deformation when external force is withdrawn.
4. Plasticity – Property by which material retains the deformation even after the withdrawal of load.
5. Ductility – Property by which a material can drawn into wires

| | |
|---|---|
| 6. Malleability – Property by which a material can be drawn in to ~~sh~~ sheets. | Any 6 |
| 7. Brittleness – It is the property by which material breaks with little permanent distortion | 6 6 |
| 8. Toughness – Property of a material to resists fracture due to impact loads. | |
| 9. Hardness – Property of a material to resists penetration, wear, scratching etc. | |
| V (b) | |
| Area of cross section of column, $A = 300 \times 300 = 90000\,mm^2$ | 1 |
| Area of steel reinforcement, $A_s = 4 \times \frac{\pi \times 20^2}{4} = 1256.64\,mm^2 = 1257\,mm^2$ | 1 |
| ∴ Area of concrete $A_c = A - A_s = 90000 - 1257 = 88743\,mm^2$. | 1 |
| Let P be the safe load for the column. | |
| Then $P = P_c + P_s$ | |
| $P = f_c \times A_c + f_s \times A_s \longrightarrow (1)$ | 1 |
| Given that, $f_c = 5\,N/mm^2$ | |
| $E_s = 15\,E_c$ | |
| ie, $\frac{E_s}{E_c} = 15 \longrightarrow (2)$. | 1 |
| Since the strain in concrete and steel are equal, $\frac{f_s}{E_s} = \frac{f_c}{E_c}$ | 1 |
| $\therefore \frac{f_s}{f_c} = \frac{E_s}{E_c}$ | |
| From (2) $\frac{f_s}{\cancel{f_c}} = 15$ | |
| $\therefore f_s = 15 \times 5$ | |
| $f_s = 75\,N/mm^2$ | 1 |
| Then from equation (1) | |
| $P = 5 \times 88743 + 75 \times 1257$ | 1 |

P6'

9

$P = 537.99 \times 10^3\ N$ 1 15

$P = 537.99\ kN.$

VI (a)

1. $E = 2N\left(1 + \frac{1}{m}\right)$, 2

2. $E = 3K\left(1 - \frac{2}{m}\right)$, 2 6

3. $E = \frac{9KN}{3K + N}$, 2

Where $E$ = modulus of elasticity

$N$ = Rigidity modulus

$K$ = Bulk modulus,

$\frac{1}{m}$ = Poisson's ratio.

VI (b)

Given that

$b = 500m$

$d = 200mm$.

$l = 10m = 10000mm$

$P = 160\ kN = 160 \times 10^3\ N$

$f_p = 200\ N/mm^2$.

$E = 200\ GPa = \frac{200 \times 10^9}{10^6}\ N/mm^2$

$= 200 \times 10^3\ N/mm^2$ Area $A = 500 \times 200 = 100000\ mm^2$

Since the load is applied suddenly

$f = \frac{2P}{A}$ 1

$= \frac{2 \times 160 \times 10^3}{100000} = 3.2\ N/mm^2.$ 1

Volume, $V = A \times l = 100000 \times 10000 = 10^9\ mm^3$ 1

i) Strain energy stored at the given rod

$= \frac{f^2}{2E} \times V$ 1

$= \frac{3.2^2}{2 \times 2 \times 10^5} \times 10^9$

$= 25600\ N\,mm$ 1

$= 25.6\ Nm$

ii) Proof resilience $= \frac{f_p^2}{2E} \times V$ 1

$$= \frac{200^2}{2 \times 2 \times 10^5} \times 10^9$$ 1

$$= 100 \times 10^6 \text{ N-mm}$$ 9

iii Modulus of resilience $= \dfrac{f_p^2}{2E}$ 1

$$= 0.1 \text{ N.mm/mm}^3.$$ 1 15

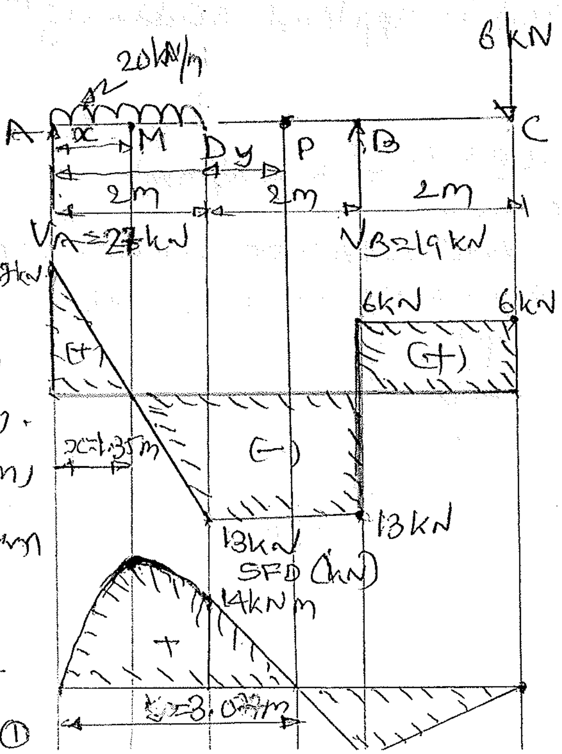
UNIT III

VII (a) i) shear force at any cross section of a beam may be defined as the unbalanced vertical forces to the right or to the left of the section. 2

ii) Bending moment at any cross section of a beam may be defined as the algebraic sum of the moment of the forces to the right or to the left of the section.

iii) Point of contraflexure is the point where the bending moment changes its sign. At point of contraflexure the BM = 0.

VII (b)




SFD → 1

BMD → 1

Let $V_A$ & $V_B$ be the support reactions at A & B respectively. Applying the conditions of static equilibrium

$\Sigma V = 0$

ie $V_A + V_B = 6 + 20 \times 2$

ie $V_A + V_B = 46 \text{ kN} \rightarrow ①$

$\Sigma M_A = 0$

$V_B \times 4 + {}^-6 \times 6 + (20 \times 2) \times 1 = 0$

$4V_B - 36 - 40 = 0$

$4V_B = 76$

$V_B = \frac{76}{4}$

$V_B = 19 kN$ | 0.5

Put $V_B = 19$ kN in equation ①

$V_A + 19 = 46$

$V_A = 46 - 19$

$V_A = 27 kN$

$V_A = 27 kN$
$V_B = 19 kN$ | 0.5

Calculation of SF

Let Right upward force as negative

SF at C = +6 kN

SF at a section just right of B = +6 kN | 1.5

SF at a section just left of B = +6 + −19 = −13 kN

SF at D = −13 kN

SF at A = −13 + 20×2 = +27 kN.

Calculation of BM

BM at C = 0

BM at B = −6×2 = −12 kNm | 1.5

BM at D = −6×4 + 19×2 = +14 kNm

BM at A = 0

Maximum BM

Let M be the point where SF changes its sign and is at a distance of 'x' from left end A. | 1

Then SF at M = 0

$27 + {}^-20 \times x = 0$

$20x = 27$

$x = \frac{27}{20} = 1.35$ m from left end A

Max BM = BM at M = $+27 \times 1.35 + {}^-20 \times 1.35 \times \frac{1.35}{2}$

$= \underline{18.225 \text{ kNm}}$ | 1

Point of contraflexure

Let P be the point when BM changes its sign and is at a distance y from ~~leftward~~ ~~Aon~~ D

Then BM at P = 0

$$27\times(y+2) + (20\times 2)(1+y) = 0$$

$$27y + 54 - 40 - 40y = 0$$

$$14 - 13y = 0$$

$$13y = 14$$

$$y = \frac{14}{13}$$ 9

$$y = 1.077\text{m from } D$$ 1

Point of contraflexure is 3.077m from left end A 15

VIII (a) i) The ratio of polar moment of inertia (J) to the radius of the shaft (R) is called the polar modulus of the section

$$Z_p = \frac{J}{R}.$$ 2

ii) The product of rigidity modulus G and polar moment of inertia J is called torsional rigidity. Torsional rigidity is the torque that produces a twist of one radian in a shaft of unit length. 2 6

iii) If the thickness of the ~~[illegible]~~ plate is negligible compared to the diameter of the shell, it is called a thin shell. For a thin shell wall thickness is usually less than or equal to 1/20 of the lateral diameter of the shell 2

VIII (b) Given that,

$t = 15\text{mm}$

$f = 120\ \text{N/mm}^2.$

$p = 4\ \text{N/mm}^2.$ $\eta_l = 0.70$, $\eta_c = 0.30$

Let $f = f_c$

Then $f_c = \dfrac{pd}{2t\eta_l}$ 1

$\therefore d = \dfrac{f_c \times 2t\eta_l}{p}$

$= \dfrac{120 \times 2 \times 15 \times 0.7}{4}$

$= 630\ mm.$ 1

Let $f = f_l$

Then $f_l = \dfrac{pd}{4t\eta_c}$ 1

$\therefore d = \dfrac{f_l \times 4t\eta_c}{p}$

$= \dfrac{120 \times 4 \times 15 \times 0.3}{4}$

$= 540\ mm.$ 1

If $d = 630\ mm$ is adopted

$f_l = \dfrac{pd}{4t\eta_c}$ 1

$= \dfrac{4 \times 630}{4 \times 15 \times 0.3}$

$= 140\ N/mm^2 > 120\ N/mm^2$

Then $f_c$ also $> 120\ N/mm^2$. 1

If $d = 540\ mm$ is adopted

$f_c = \dfrac{pd}{2t\eta_l}$

$= \dfrac{4 \times 540}{2 \times 15 \times 0.7}$

$= 102.86\ N/mm^2 < 120\ N/mm^2$. 1

Then $f_l$ also $< 120\ N/mm^2$. 1

Thus if $d = 540\ mm$ is adopted both $f_c$ and $f_l$ values will remain within safe limit of stress in material. 9

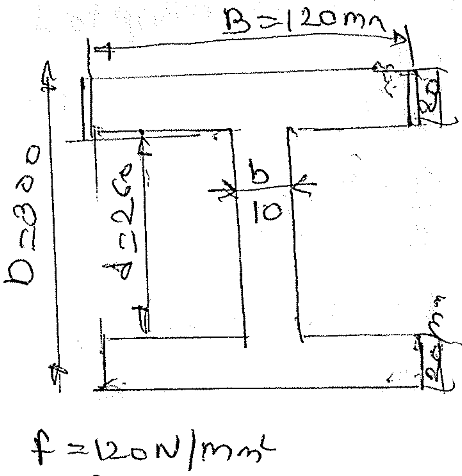
$\therefore$ Required diameter $= 540\ mm.$ 1

## UNIT IV

IX (a)

1. The beam is subjected to pure bending and is therefore free from shear force.
2. The material of the beam is homogeneous and isotropic
3. Each longitudinal fibre is free to expand or contract independently.
4. The value of the Young's modulus is the same for beam material in tension and as well as compression. — Any 6
5. A transverse section of the beam which is a plane before bending remain a plane after bending. — 6 — 6
6. The elastic limit is not exceeded.
7. The resultant thrust or pull on a transverse section of the beam is zero.
8. The radius of curvature of the deflected beam is very large compared with the dimensions of the cross section of the beam.
8. The transverse section of the beam is symmetrical about an axis passing through the centroid of the section and parallel to the plane of bending.

IX (b)




Given that

$B = 120\,mm$

$D = 300\,mm$

$t_f = 20\,mm$

$b = t_w = 10\,mm$ — 1

$d = 300 - 2t_f$

$= 300 - 20 - 20$

$= 260\,mm$

$l = 5m = 5000\,mm$, $f = 120\,N/mm^2$

Moment of Inertia $I = \dfrac{BD^3}{12} - \dfrac{(B-b)\,d^3}{12}$ — 1

$= \dfrac{120 \times 300^3}{12} - \dfrac{(120-10)\,260^3}{12}$

$= 108.887 \times 10^6\,mm^4$ — 1

Depth of neutral axis (y)

$y = \frac{D}{2} = \frac{300}{2} = 150\,mm$ 1

Section modulus,

$Z = \frac{I}{y}$ 1

$= \frac{108.887 \times 10^6}{150}$

$= 725.911 \times 10^3\ mm^3$ 1

$w = ?$

For a simply supported beam with udl

Maximum BM $= M = \frac{wl^2}{8}$. 1

$M = fZ$. 1

$\frac{wl^2}{8} = fZ$

$w = \frac{fZ \times 8}{l^2}$

$= \frac{120 \times 725.911 \times 10^3 \times 8}{5000^2}$ 9

$w = 27.87\ N/mm$. 1 15

$w = 27.87\ kN/m$

X (a) i) The neutral axis of any transverse section of a beam is defined as the line of intersection of the neutral layer with the transverse section 2

ii Section modulus is defined as the ratio of moment of inertia (I) about the neutral axis (...) to the distance (y) of the extreme fibre of the cross section from the neutral axis. 2

ie, $Z = \frac{I}{y_{max}}$.

iii) The product of Young's modulus E and moment of inertia I is known as flexural rigidity. 2 6

EI is called flexural rigidity.

X (b)

Given that

$b = 100mm$

$d = 300mm.$

$F = 60kN = 60 \times 10^3 N$

$A = b \times d = 100 \times 300 = 30000 mm^2.$

i) Average shear stress

$\tau_{avg} = \frac{F}{A}$

$= \frac{60 \times 10^3}{30000}$ 3

$= 2 N/mm^2.$

ii) Maximum shear stress

$\tau_{max} = 1.5 \tau_{avg}$

$\tau_{max} = \frac{3}{2} \frac{F}{bd}$ 3

$= 1.5 \tau_{avg}$

$= 1.5 \times 2$

$= 3 N/mm^2.$

iii) Shear stress at a distance of 40mm above neutral axis 9

$y = 40mm.$

$I = \frac{bd^3}{12} = \frac{100 \times 300^3}{12} = \underline{225 \times 10^6 mm^4}$ 3

$\tau = \frac{F}{2I}\left(\frac{d^2}{4} - y^2\right)$

$= \frac{60 \times 10^3}{2 \times 225 \times 10^6}\left(\frac{300^2}{4} - 40^2\right)$ 15

$= 2.787 N/mm^2$