Scoring Indicators

Course Code: **2002** ENGINEERING MATHEMATICS Revision: 2015

Qn. No.	Scoring Indicators	Split Score	Sub Total	Total
	Part A			
Q I.1	unit vector $= \frac{2i+j+2k}{\sqrt{2^2+1+2^2}} = \frac{2i+j+2k}{3}$	2	2	
Q I.2			2	
	$\begin{vmatrix} x & 16 \\ 4 & x \end{vmatrix} = x^2 - 64 = 0$	1		
	$\Rightarrow x = \pm 8$	1		
Q I.3	$5\begin{bmatrix} 1 & 2 \\ -2 & 0 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 5 & -7 \end{bmatrix} = \begin{bmatrix} 5 & 10 \\ -10 & 0 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 5 & -7 \end{bmatrix}$	1	2	
	$= \begin{bmatrix} 2 & 10 \\ -15 & 7 \end{bmatrix}$	1		10
Q I.4	$\int x^2(x+1)dx = \int x^3dx + \int x^2dx$	1		
	$= \frac{x^4}{4} + \frac{x^3}{3} + C$	1	2	
Q I.5	$\frac{dy}{dx} = 2y \Rightarrow \frac{dy}{y} = 2dx \Rightarrow logy = 2x + \log C$	1		
	$y = Ce^x$	1	2	

	Part B			
Q II.1	Vector perpendicular to the vectors $\vec{a}$ and $\vec{b}$ is $\vec{a}\times\vec{b}$	2		
	$\vec{a}\times\vec{b}=\begin{vmatrix} i & j & k \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{vmatrix}=-4i+2j+2k$	2		
	unit vector is given by $\dfrac{-4i+2j+2k}{\sqrt{(-4)^2+2^2+2^2}}=\dfrac{-4i+2j+2k}{\sqrt{24}}$		6	
Q II.2	$(r+1)^{\text{th}}$ term= $15C_r(x^3)^{15-r}\left(\frac{3}{x^2}\right)^r = 15C_r3^rx^{45-5r}$	2		
	$45-5r=0\Rightarrow r=9$	3		
	Constant term is $15C_93^9$	1	6	
QII. 3	$2A=(A+B)+(A-B)=\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}+\begin{bmatrix} 5 & 4 & 3 \\ 2 & 1 & 0 \end{bmatrix}=\begin{bmatrix} 6 & 6 & 6 \\ 6 & 6 & 6 \end{bmatrix}$	2		
	$A=\begin{bmatrix} 3 & 3 & 3 \\ 3 & 3 & 3 \end{bmatrix}$	1		
	$2B=(A+B)-(A-B)=\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}-\begin{bmatrix} 5 & 4 & 3 \\ 2 & 1 & 0 \end{bmatrix}=\begin{bmatrix} -4 & -2 & 0 \\ 2 & 4 & 6 \end{bmatrix}$	2		
	$B=\begin{bmatrix} -2 & -1 & 0 \\ 1 & 2 & 3 \end{bmatrix}$	1		
Q II.4	$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 3 \\ 1 & 4 & 9 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} 1 \\ 6 \\ 3 \end{bmatrix}$	1		
	$A=\begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 3 \\ 1 & 4 & 9 \end{bmatrix}$, $\det(A)=\begin{vmatrix} 1 & 1 & 1 \\ 2 & 2 & 3 \\ 1 & 4 & 9 \end{vmatrix}=-3$	2	6	
	$A^{-1}=\dfrac{1}{-3}\begin{bmatrix} 6 & -5 & 1 \\ -15 & 8 & -1 \\ 6 & -3 & 0 \end{bmatrix}$	2		30
	$x=7,\quad y=-10,\quad z=4$	1		
Q II.5	$\int(x+\frac{1}{x})^2\,dx=\int(x^2+\frac{1}{x^2}+2)\,dx$	2		
	$=\int x^2dx+\int\frac{1}{x^2}dx+2\int dx$	2	6	

	$=\dfrac{x^3}{3}-\dfrac{1}{x}+2x+C$	2		
QII.6	$\int_0^{\pi} cos^2xdx=\int_0^{\pi}\dfrac{1+cos2x}{2}dx$	2		
	$=\dfrac{1}{2}\left[x+\dfrac{sin2x}{2}\right]_0^{\pi}$	2	6	
	$=\dfrac{\pi}{4}$	2		
QII.7	$\dfrac{dy}{dx}+\dfrac{y}{(1+x^2)}=\dfrac{e^{tan^{-1}(x)}}{(1+x^2)}$	1		
	$P=\dfrac{1}{(1+x^2)},Q=\dfrac{e^{tan^{-1}(x)}}{(1+x^2)}$	1		
	$IF= e^{\int\frac{1}{(1+x^2)}dx}=e^{tan^{-1}(x)}$	1	6	
	$solution\ is\ given\ by\ ye^{tan^{-1}(x)}=\int\dfrac{e^{tan^{-1}(x)}}{(1+x^2)}e^{tan^{-1}(x)}\,dx$	1		
	put $u=e^{tan^{-1}(x)}, du=\dfrac{1}{(1+x^2)}\,dx$	1		
	$ye^{tan^{-1}(x)}=\dfrac{\left[e^{tan^{-1}(x)}\right]^2}{2}+C$	1		

	PART C **MODULE 1**			
Q III.1	$\vec{a}.\vec{b} = (i-2j+3k).(3i-2j+k) = 3+4+3 = 10$	2		
	$\theta = cos^{-1}\left[\frac{\vec{a}.\vec{b}}{\overrightarrow{\lvert ab \rvert}}\right]$	1		
		1	5	
	$\overrightarrow{\lvert ab \rvert} = \left(\sqrt{1^2+-2^2+3^2}\right)\left(\sqrt{3^2+-2^2+1^2}\right) = \sqrt{14}\sqrt{14} = 14$	1		
	$\theta = cos^{-1}\left[\frac{10}{14}\right]$	1		
Q III.2	Given $\vec{F} = 4i-3k$ and $\vec{r} = \overrightarrow{BA}$			
		1		
	$\vec{r} = (2i-2j+5k)-(i-3j+k) = i+j+4k$	2	5	
	$m = \lvert\vec{r}\times\vec{F}\rvert = \begin{vmatrix} i & j & k \\ 1 & 1 & 4 \\ 4 & 0 & -3 \end{vmatrix} = \lvert -3i+19j-4k \rvert = \sqrt{386}$	2		15
QIII. 3	$(r+1)^{th}$ term $= 10C_r(x^2)^{10-r}\left(\frac{2}{x}\right)^r$	1		
	7^{th} term $= (6+1)^{th}$ term $= 10C_6(x^2)^{10-6}\left(\frac{2}{x}\right)^6$	2	5	
	$= 10C_6(x^2)^4\left(\frac{2}{x}\right)^6 = 10C_6x^2\,2^6$	2		
	OR			
Q IV.1	$\vec{a}+\vec{b} = (5i-j-3k)+(i+3j-5k) = 6i+2j-8k$	1		
	$\vec{a}-\vec{b} = (5i-j-3k)-(i+3j-5k) = 4i-4j+2k$	1		
	$(\vec{a}+\vec{b}).(\vec{a}-\vec{b}) = (6i+2j-8k).(4i-4j+2k) = 24-8-16 = 0$	2	5	
	$(\vec{a}+\vec{b})$ and $(\vec{a}-\vec{b})$ are perpendicular.	1		
Q IV.2	$\overrightarrow{AB} = (2i+j+5k)-(i-k) = i+j+6k$			
	$\overrightarrow{AC} = (j+2k)-(i-k) = -i+j+3k$	1		
		1		15

	Area of $\Delta ABC = \frac{1}{2}\lvert\overrightarrow{AB} \times \overrightarrow{AC}\rvert = \begin{vmatrix} i & j & k \\ 1 & 1 & 6 \\ -1 & 1 & 3 \end{vmatrix} = \frac{1}{2}\lvert -3i + 19j - 4k \rvert$	2	5	
QIV.3	$= \frac{1}{2}\sqrt{-3^2 + -9^2 + 2^2} = \frac{\sqrt{94}}{2}$	1		
	$(r+1)^{\text{th}}$ term $= 9C_r(2x)^{9-r}\left(\frac{3}{x}\right)^r$			
	Since $n =$ 9, odd, there are two middle terms $(\frac{n+1}{2})^{\text{th}}$ and $(\frac{n+3}{2})^{\text{th}}$ terms	1		
	5^{th} term $= (4+1)^{\text{th}}$ term $= 9C_4(2x)^{9-4}\left(\frac{3}{x}\right)^4$		5	
	$= 9C_4(2x)^5\left(\frac{3}{x}\right)^4 = 9C_4 2^5 3^4 x$	2		
	6^{th} term $= (5+1)^{\text{th}}$ term $= 9C_5(2x)^{9-5}\left(\frac{3}{x}\right)^5$	2		
	$= 9C_5(2x)^4\left(\frac{3}{x}\right)^5 = 9C_4 2^4 \frac{3^5}{x}$			
	MODULE II			
Q V. 1	$\begin{vmatrix} 2 & 1 & x \\ 3 & -1 & 2 \\ 1 & 1 & 6 \end{vmatrix} = 2(-6-2) - 1(18-2) + x(3+1) = 4x - 32$	2	5	
	$\begin{vmatrix} 4 & x \\ 3 & 2 \end{vmatrix} = 8 - 3x$	2		
	$4x - 32 = 8 - 3x \Rightarrow 7x = 40 \Rightarrow x = \frac{40}{7}$	1		
Q V. 2	$AB = \begin{bmatrix} 5 & 3 \\ 2 & 2 \end{bmatrix}\begin{bmatrix} 7 & 5 \\ 4 & 3 \end{bmatrix} = \begin{bmatrix} 47 & 34 \\ 22 & 16 \end{bmatrix}, (AB)^{-1} = \frac{1}{4}\begin{bmatrix} 16 & -34 \\ -22 & 47 \end{bmatrix}$	2		15
	$A^{-1} = \frac{1}{4}\begin{bmatrix} 2 & -3 \\ -2 & 5 \end{bmatrix}$	1	5	
	$B^{-1} = \frac{1}{1}\begin{bmatrix} 3 & -5 \\ -4 & 7 \end{bmatrix}$	1		
Q V. 3	$B^{-1}A^{-1} = \begin{bmatrix} 3 & -5 \\ -4 & 7 \end{bmatrix}\frac{1}{4}\begin{bmatrix} 2 & -3 \\ -2 & 5 \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 16 & -34 \\ -22 & 47 \end{bmatrix} \therefore (AB)^{-1} = B^{-1}A^{-1}$	1		
		2		
			5	

	$\Delta = \begin{vmatrix} 1 & 2 & -1 \\ 3 & 1 & 1 \\ 1 & -1 & 2 \end{vmatrix} = -3$ $\Delta_1 = \begin{vmatrix} -3 & 2 & -1 \\ 4 & 1 & 1 \\ 6 & -1 & 2 \end{vmatrix} = -3$	2		
	$\Delta_2 = \begin{vmatrix} 1 & -3 & -1 \\ 3 & 4 & 1 \\ 1 & 6 & 2 \end{vmatrix} = 3\,, \Delta_3 = \begin{vmatrix} 1 & 2 & -3 \\ 3 & 1 & 4 \\ 1 & -1 & 6 \end{vmatrix} = -6$	1		
	$x = \frac{\Delta_1}{\Delta} = \frac{-3}{-3} = 1,\ \ y = \frac{\Delta_2}{\Delta} = \frac{3}{-3} = -1,\ \ z = \frac{\Delta_3}{\Delta} = \frac{-6}{-3} = 2$	3		
Q VI. 1	**OR**		5	
Q VI.2	$AB = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \begin{bmatrix} 1 & 4 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 4 & -1 \\ 2 & 8 & -2 \\ 3 & 12 & -3 \end{bmatrix}$	2		
	$BA = \begin{bmatrix} 1 & 4 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = 1 + 8 - 3 = 6$	2		
			5	15
	$(A + A^T)^T = A^T + (A^T)^T = A^T + A = A + A^T, \quad \therefore A + A^T$ is symmetric	3		
Q VI.3	$(A - A^T)^T = A^T - (A^T)^T = A^T - A = -(A - A^T),$	2		
	$\therefore A - A^T$ is skew symmetric		5	
	$A = \begin{bmatrix} 3 & 1 & -1 \\ -1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}, \lvert A \rvert = \begin{vmatrix} 3 & 1 & -1 \\ -1 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix} = 4$	3		
Q VII.1	Adjoint matrix $= \begin{bmatrix} 0 & -2 & 2 \\ 4 & 4 & -2 \\ -2 & -2 & 4 \end{bmatrix}, A^{-1} = \frac{1}{4} \begin{bmatrix} 0 & -2 & 2 \\ 4 & 4 & -2 \\ -2 & -2 & 4 \end{bmatrix}$	1		
	MODULE III			
	$\int cosecx\, dx = \int \frac{cosecx(cosec\, x - \cot x)}{cosec\, x - \cot x}\, dx$	1		
	$= \int \frac{cosec^2 x - cosec\, x \cot x}{cosec\, x - \cot x}\, dx$	1	5	
		1		
	let u $= cosec\, x - \cot x,\ \ du = cosec^2 x - cosec\, x \cot x\, dx$	1		
Q VII.2	$\int \frac{cosec^2 x - cosec\, x \cot x}{cosec\, x - \cot x}\, dx = \int \frac{du}{u} = \log u + C$	2		
	$= \log(cosec\, x - \cot x) + C$	1	5	15

Q VII.3	$\int_0^{\pi} \frac{1}{1+sinx}dx = \int_0^{\pi} \frac{1-sinx}{1+sinx(1-sinx)}dx = \int_0^{\pi} \frac{1-sinx}{1-sin^2x}dx$ $= \int_0^{\pi} \frac{1-sinx}{cos^2x}dx = \int_0^{\pi} \frac{1}{cos^2x}dx - \int_0^{\pi} \frac{sinx}{cos^2x}dx$ $= \int_0^{\pi} \sec^2 x\, dx - \int_0^{\pi} \tan x \sec x\, dx = [\tan x - \sec x]_0^{\pi}$ $= \tan\pi - \sec\pi - (\tan 0 - \sec 0) = 2$	1 1 2 2 1	5	
QVIII.1	$\int xsinx\, dx = x\int \sin x\, dx - \int \left(\frac{d(x)}{dx}\int \sin x\, dx\right) dx$ $= -xcos\, x + \int \cos x\, dx$ $= -xcos\, x + \sin x + C$ **OR**	1 1 1	5	
QVIII.2	$\int \frac{1+cosx}{(x+sinx)^2}dx$ Put $t = x + sinx,\ \frac{dy}{dt} = 1 + cosx$ $\int \frac{1+cosx}{(x+sinx)^2}dx = \int \frac{dt}{t^2}$ $= -\frac{1}{t} + C$ $= -\frac{1}{x+sinx} + C$	1 1 2 1 1	5	15
QVIII.3	$\int x^2e^{-x}dx = x^2\int e^{-x}\, dx - \int \left(\frac{d(x^2)}{dx}\int e^{-x}\, dx\right) dx$ $= x^2\frac{e^{-x}}{-1} - \int \left(2x\frac{e^{-x}}{-1}\right) dx = -x^2e^{-x} + 2\int xe^{-x}dx$ $= -x^2e^{-x} + 2[-xe^{-x} - \int \frac{d}{x}(x)\int e^{-x}\, dx$ $= -x^2e^{-x} - 2xe^{-x} + 2e^{-x} + C$ Put $t = logx,\ \frac{dt}{dx} = \frac{1}{x}$	1 1 2 1 1	5	

	$\int_1^e \frac{\sin(\log x)}{x}dx = \int_1^e \sin t\, dt$			
Q IX.1	$= -cost + C$			
		3		
	$= [-\cos(\log x)]_1^e = 1 - \cos 1$		5	
	MODULE IV			
		2		
Q IX. 2	Given $y = 2\cos x$			
	Volume $V = \pi\int_a^b y^2dx = \pi\int_0^{\frac{\pi}{4}} 4cos^2 x dx = \pi\int_0^{\frac{\pi}{4}}\frac{1+cos2x}{2}dx$	1		
		1		
	$= \frac{4\pi}{2}\left[x + \frac{\sin 2x}{2}\right]_0^{\frac{\pi}{4}} = \frac{4\pi}{2}\left[\frac{\pi}{4} + \frac{1}{2}\right]$ cubic units	1	5	15
	$P = \cot x, Q = 2\cos x$	1		
	$IF = e^{\int Pdx} = e^{\int \cot x dx} = e^{\log(\sin x)} = \sin x$	1		
	Solution is given by ysin x $= \int 2\cos x \sin x dx + C$			
Q IX.3				
		2		
	ysin x $= \int sin2xdx + C = \frac{-cos\,2x}{2} + C$		5	
	$y = \frac{-cos\,2x}{2sinx} + \frac{C}{\sin x}$	2		
		1		
	$\frac{3e^x}{(1-e^x)}dx + \frac{sec^2y}{tany}dy = 0 \Rightarrow \frac{3e^x}{(1-e^x)}dx = -\frac{sec^2y}{tany}dy$			
Q X.1				
	$\int\frac{3e^x}{(1-e^x)}dx = -\int\frac{sec^2y}{tany}dy = -3\log(1-e^x) = -\log(\tan y) - \log C$	2		
			5	
	$(1-e^x)^3 = Ctan\, y$			
		3		
	OR			
Q X. 2		1		
	At the point of intersection $4 - y^2 = 0 \Rightarrow y = \pm 2$			
	Area $= \int_{-2}^{2}(4-y^2)dy = \left[4y - \frac{y^3}{3}\right]_{-2}^{2} = 16 - \frac{16}{3} = \frac{32}{3}$	2	5	
		2		
	$dy = e^{3x}e^ydx = \frac{1}{e^y}dy = e^{3x}dx$			

Q X. 3	$\Rightarrow \int \frac{1}{e^y} dy = \int e^{3x} dx$	1		
	$-e^{-y} = \frac{e^{3x}}{3} + C$	1	5	
	$\frac{dy}{dx} + 3\frac{y}{x} = 5x$	1		
	$P = \frac{3}{x}, Q = 5x$	1		
	$IF = e^{\int P dx} = e^{\int \frac{3}{x} dx} = e^{3\log x} = x^3$	1		
	Solution is given by $yx^3 = \int xx^3 dx + C = \int x^4 dx + C$			
	$yx^3 = \frac{x^4}{4} + C \Rightarrow y = \frac{x}{4} + \frac{C}{x^3}$			