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Scoring Indicators

Course Code: **1002** ENGINEERING MATHEMATICS I Revision: 2015

Qn. No.	Scoring Indicators	Split Score	Sub Total	Total
	Part A			
Q I.1	$sin^2\theta + cos^2\theta + 2sin\theta cos\theta$	1		
	$1 + sin2\theta$	1	2	
Q I.2	$sin(60 + 30) = sin(90) = 1$	2	2	
Q I.3	Area $= \frac{1}{2} \times 4 \times 2 \times \frac{1}{2} = 2$	2	2	10
Q I.4	$x^2 \times \frac{1}{x} + 2xlogx = \ x + 2xlogx$	2	2	
Q I.5	$\frac{dy}{dx} = 2x - 3 = 3$	2	2	

Q	**Part B**			
II.1	$\sqrt{3}\cos x + \sin x = R\sin x\cos\alpha + R\cos x\sin\alpha$	1		
	$\sqrt{3} = R\sin\alpha, \quad 1 = R\cos\alpha$	1		
	$R = \pm 2$	2	6	
	$\sqrt{3} = \tan\alpha$	1		
	$\sqrt{3}\cos x + \sin x = \pm 2\sin(x + 60^0)$	1		
Q II.2	$\sin 20°(-\frac{1}{2}(\cos 120° - \cos 40^0))$	1		
	$\sin 20°(-\frac{1}{2}(-\frac{1}{2} - \cos 40^0))$	1		
	$\frac{1}{4}\sin 20^0 + \frac{1}{2}(\sin 20^0 \cos 40^0)$	2	6	
	$\frac{1}{4}\sin 20^0 + (\frac{1}{4}(\sin 60^0 - \sin 20^0) = \frac{1}{8}$	2		
QII. 3	$\frac{dy}{dx} = -\frac{a\sin(\log x)}{x} + \frac{b\cos(\log x)}{x}$	2		
	$\frac{d^2y}{dx^2} = \frac{-(a\cos(\log x) + b\sin(\log x)) + \frac{a\sin(\log x)}{x} - \frac{b\cos(\log x)}{x}}{x^2}$	2	6	
	$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$			
				30
Q II.4	$volume\ V = (18 - 2x)(18 - 2x)x = 4x^3 - 72x^2 + 324x$	2		
	$\frac{dv}{dx} = 12x^2 - 144x + 324 = 12(x - 9)(x - 3) = 0$	2		
	$\frac{d^2v}{dx^2}_{x=3} < 0$, Area is maximum at $x = 3$	2		

Q II.5	$A = cos^{-1}\left[\frac{b^2+c^2-a^2}{2bc}\right] = 34^0 03'$	2		
	$B = cos^{-1}\left[\frac{a^2+c^2-b^2}{2ac}\right] = 44^0 25'$	2	6	
	$C = cos^{-1}\left[\frac{b^2+a^2-c^2}{2ba}\right] = 101^0 32'$	2		
QII.6	Figure	2		
	Width of the river is 20m	2		
	Height of the tree is $20\sqrt{3}$	2		
QII.7	$\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x} = \lim_{\Delta x\to 0}\frac{-sin\left(x+\frac{\Delta x}{2}\right)sin\left(\frac{\Delta x}{2}\right)}{\frac{\Delta x}{2}}$	4	6	
	$= \lim_{\Delta x\to 0} - sin\left(x+\frac{\Delta x}{2}\right)\lim_{\Delta x\to 0}\frac{sin\left(\frac{\Delta x}{2}\right)}{\frac{\Delta x}{2}} = -sinx$	2		

	PART C **MODULE 1**			
Q III.1	$\frac{\sec\theta}{\sec\theta+1}+\frac{\sec\theta}{\sec\theta-1}=\frac{\sec^2\theta-sec\theta+\sec^2\theta+\sec\theta}{\sec^2\theta-1}$	3		
	$=\frac{2sec^2\theta}{tan^2\theta}=2cosec^2\theta$	2	5	
Q III.2	$\frac{tan45-tan30}{1+tan45tan30}=\frac{1-\frac{1}{\sqrt{3}}}{1+1\frac{1}{\sqrt{3}}}=\frac{\sqrt{3}-1}{\sqrt{3}+1}$	3	5	15
	$\frac{\sqrt{3}-1}{\sqrt{3}+1}=2-\sqrt{3}$	2		
QIII. 3	**Figure**			
	Distance between the base of the light house and first boat =	2		
	$\frac{90}{\sqrt{3}}\,m$	2	5	
	Distance between the boats = 38.1 m	1		
Q IV.1	$\sqrt{\frac{1-sin\theta}{1+sin\theta}}=\sqrt{\frac{1-sin\theta}{1+sin\theta}}\sqrt{\frac{1-sin\theta}{1-sin\theta}}$	2		
	$=\frac{1-sin\theta}{cos\,\theta}$	2	5	
	$\frac{1}{cos\theta}-tan\theta=sec\theta-tan\theta$	1		
Q IV.2	$tan(A+B)=\frac{tan\,A+tan\,B}{1-tan\,Atan\,B}$	1		
	$=\frac{\frac{1}{2}+\frac{1}{3}}{1-\frac{1}{2}\frac{1}{3}}=1$	2	5	
	$A+B=tan^{-1}(1)=45$	2		15

QIV.3	$sin\left(\frac{\pi}{3}+A\right)-sin\left(\frac{\pi}{3}-A\right)$			
	$=sin\frac{\pi}{3}cos\,A+cos\frac{\pi}{3}sinA-\left(sin\frac{\pi}{3}cos\,A-cos\frac{\pi}{3}sinA\right)$	3		
			5	
	$=2cos\frac{\pi}{3}sinA=2\times\frac{1}{2}sinA=SinA$			
		2		
	$\cos^4 A-\sin^4 A=(\cos^2 A-\sin^2 A)(\cos^2 A+\sin^2 A)$	2		
	$=\cos^2 A-\sin^2 A$	1	5	
	$=\cos A\cos A-\sin A\sin A=\cos(A+A)=\cos 2A$	2		
Q V. 1	$\tan(50-40)=\frac{\tan 50-\tan 40}{1+\tan 50\tan 40}$	1		
	$=\frac{\tan 50-\tan 40}{1+\tan 50\tan(90-50)}=\frac{\tan 50-\tan 40}{1+\tan 50\cot 50}$	2	5	
	$\tan 10=\frac{\tan 50-\tan 40}{2}\Rightarrow 2\tan 10+\tan 40=\tan 50$	2		
Q V. 2	Figure, $B=53^0 17', C=67^0 09', a=100\ cm$			
	$A=180-(B+C)=59^0 34'$			
		2		
	$b=\frac{\sin 53^0 17'\times 100}{\sin 59^0 34'}=92.97m$	1	5	
	$c=\frac{\sin 67^0 09'\times 100}{\sin 59^0 34'}=92.97m$	1		15
		1		
Q V. 3	Required fencing length is $a+b+c=299.88$			
	$\frac{\sin A+\sin 3A+\sin 5A}{\cos A+\cos 3A+\cos 5A}$			
	$=\frac{2\sin\left(\frac{A+5A}{2}\right)\cos\left(\frac{A-5A}{2}\right)+\sin 3A}{2\cos\left(\frac{A+5A}{2}\right)\cos\left(\frac{A-5A}{2}\right)+\cos 3A}$	3		
			5	
	$=\frac{\sin 3A\,(1+\cos 2A)}{\cos 3A(1+\cos 2A)}=\frac{\sin 3A}{\cos 3A}=\tan A$	2		

Q VI. 1	$cos3A + cos5A + cos9A + cos17A$			
	$= 2\cos\left(\frac{3A+5A}{2}\right)\cos\left(\frac{3A-5A}{2}\right)$			
	$+ 2\cos\left(\frac{9A+17A}{2}\right)\cos\left(\frac{9A-17A}{2}\right)$	2		
Q VI.2	$= 2\cos 4A \cos A + 2\cos 13 \cos 4A = 2\cos 4A(\cos A + \cos 13A)$			
		1	5	
	$= 2\cos A\left(2\cos\left(\frac{A+13A}{2}\right)\cos\left(\frac{A-13A}{2}\right)\right)$	2		
	$= 4\cos 4A \cos 6A \cos 7A$			15
Q VI.3				
	$abc(\cot A + \cot B + \cot C) = abc\left(\frac{\sin A}{\cos A} + \frac{\sin B}{\cos B} + \frac{\sin C}{\cos C}\right)$	1		
	$= \frac{a}{\sin A} bc\cos A + \frac{b}{\sin B} ac\cos B + \frac{c}{\sin C} ab\cos C$	2	5	
	$= 2R\left[\frac{b^2+c^2-a^2}{2} + \frac{a^2+c^2-b^2}{2} + \frac{a^2+b^2-c^2}{2}\right] = R(a^2+b^2+c^2)$	2		
Q VII.1				
	Put $x = c + h, as\ x \to c, h \to 0$	2		
	$\lim\limits_{x\to c}\frac{x^n - c^n}{x - c} = \lim\limits_{h\to 0}\frac{(c+h)^n - c^n}{h}$			
	$= \frac{c^n + {}^nC_1 c^{n-1}h + \cdots + {}^nC_n h^n - c^\wedge n}{h}$	2	5	
		1		
Q VII.2	$= {}^n C_1 c^{n-1} = nc^{n-1}$			
				15
	$\frac{dy}{dx} = \frac{\frac{d}{dt}(b(sint - tcost))}{\frac{d}{dx}(a(cost + tsint))} = \frac{b(\cos t + t\sin t - \cos t)}{a(\cos t + t\sin t - \cos t)}$	3		
			5	
	$= \frac{bt\sin t}{at\cos t} = \frac{b}{a}\tan t$	2		
Q VII.3				
	$\frac{d}{dx}(x^2 + xy + y^2) = \frac{d}{dx}x^2 + \frac{d}{dx}xy + \frac{d}{dx}y^2 = 0$	1		
		2		
	$= 2x + y + x\frac{dy}{dx} + 2y\frac{dy}{dx} = 0$			
			5	

	$\frac{dy}{dx} = \frac{-(2x+y)}{x+2y}$	2		
QVIII.1	i) $\frac{d}{dx}\left(e^{(\cos x + 2x^2)}\right) = e^{(\cos x+2x^2)}\frac{d}{dx}(2x^2)$	1		
	$= 4xe^{(\cos x+2x^2)}$	1		
	ii) $\frac{d}{dx}\left(x^2(1+\cos x)\right) = x^2\frac{d}{dx}(1+\cos x) + (1+\cos x)\frac{d}{dx}(x^2)$	1	5	
	$= -x^2\sin x + 2x + \cos 2x$	2		
QVIII.2	$\frac{dy}{dx} = \frac{\frac{d}{d\theta}(b\tan\theta)}{\frac{d}{d\theta}(a\sec\theta)} = \frac{b\sec^2\theta}{a\sec\theta\tan\theta}$	3	5	15
	$= \frac{b\sec\theta}{a\tan\theta} = \frac{b}{a}\operatorname{cosec}\theta$	2		
QVIII.3	$y' = 1 - \frac{1}{x^2}$	2		
	$y'' = \frac{2}{x^3}$	2	5	
	$x^2y'' + xy' = y \Rightarrow x^2\frac{2}{x^3} + x\left(1 - \frac{1}{x^2}\right) = x + \frac{1}{x} = y$	1		
Q IX.1	$Velocity, v = \frac{d}{dt}\left(\frac{2}{3}t + \cos t\right) = \frac{2}{3} - \sin t$	3	5	
	$Acceleration, a = -\cos t$	2		
Q IX. 2	$Volume, V = \frac{4}{3}\pi r^3, \quad \frac{dV}{dt} = -10cc/sec$	1		
	$\frac{dV}{dt} = \frac{4}{3}\pi 3r^2\frac{dr}{dt}$	1		
	$\frac{dr}{dt} = -\frac{1}{90\pi}$	1		
	$Surface\ area\ S = 4\pi r^2$	1	5	
	$\frac{dS}{dt} = 4\pi 2r\frac{dr}{dt} = -\frac{4}{3}\ cm^2/sec$	1		
		1		

Q IX.3	$\frac{d}{dx}\left(2x^3 - 9x^2 + 12x + 2\right) = 6x^2 - 18x + 12 = 6(x^2 - 3x + 2)$			
		1		
	$\frac{dy}{dx} = 0 \Rightarrow x = 2, 1$	1		15
		1		
	$\frac{d^2}{dx^2}\left(2x^3 - 9x^2 + 12x + 2\right) = 12x - 18$			
		1	5	
	$\frac{d^2}{dx^2}_{x=1} = -6 < 0 \qquad \frac{d^2}{dx^2}_{x=2} = 6 > 0$			
		1		
	minimum value is at $x = 2$ *and is* 6			
Q X.1				
	$Area, A = \pi r^2, \frac{dr}{dt} = 2\ inch/sec$	1		
	$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$	2	5	
		1		
	Radius at the end of 3 min is $r = 3 + 3 \times 2 = 9\ inch$			
	$\frac{dA}{dt} = 2\pi \times 9 \times 2 = 36\pi inch^2/sec$	1		
Q X. 2	$\frac{dy}{dx} = \frac{1 - x^2}{(1 - x)^4}$	2		
			5	
	Tangent is parallel to the x-axis, $\frac{dy}{dx} = 0$	1		15
	$\frac{(1 + x)(1 - x)}{(1 - x)^4} = \frac{(1 + x)}{(1 - x)^3} = 0 \Rightarrow 1 + x = 0, x = -1$	2		
Q X. 3	$2x + 2y = 100$			
		1	5	
	$y = \frac{100 - 2x}{2}$			
		2		
	Area $A = x \times y = \left(\frac{100-2x}{2}\right) = \frac{1}{2}(100x - 2x^2)$			
	$\frac{dA}{dx} = 0 \Rightarrow \frac{1}{2}(100 - 4x) = 0 \Rightarrow x = 25, y = 25$			
		2	5	
	$\frac{d^2A}{dx^2}_{x=25} < 0$	1		