242 (D) April-24 PageNO-22

SCHEME OF VALUATION

Course Title : 4014-Theory of Structures II

Qn. No.	Scoring Indicator	Split-up Score	Sub Total	Total
	PART A			
1)	It is defined as the ratio b/w the equivalent length of the column to the least radius of gyration.	02	02	
2)	When the line of action of load does not coincide with the axis of the column, the load is known as eccentric load.	02	02	
3)	Fixed beams, Continuous beams, 2-hinged arch, multi storied frames.	4x½	02	
4)	Beam is designed to resist the bending and shear stresses developed in the beam. This is ensured by choosing the cross section such that the maximum bending and shear stresses developed are within permissible limits.	02	02	
5)	The moment induced at the far end due to the distributed moment at the nearer end, the far end being fixed is known as carry over moment.	02	02	10
	PART - B			
1)	$P_{cr_1} = 25$ kN $\quad L_e = L/2$ $P_{cr_2} = ?$ $\quad L_e = L/\sqrt{2}$.	01		
	$P_{cr} = \frac{\pi^2 EI}{L_e^2} \Rightarrow P_{cr} \propto \frac{1}{L_e^2}$	02		

$$P_{CR2} = \frac{25 \times (L/2)^2}{(L/\sqrt{2})^2} = 12.5 \text{ kN}$$

03 06

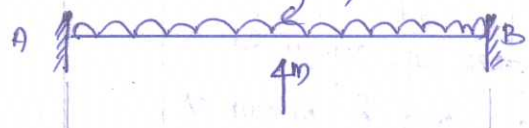
2) Perfect frames : Equation $m = 2j - 3$ is satisfied

Deficient frames : $m < 2j - 3$

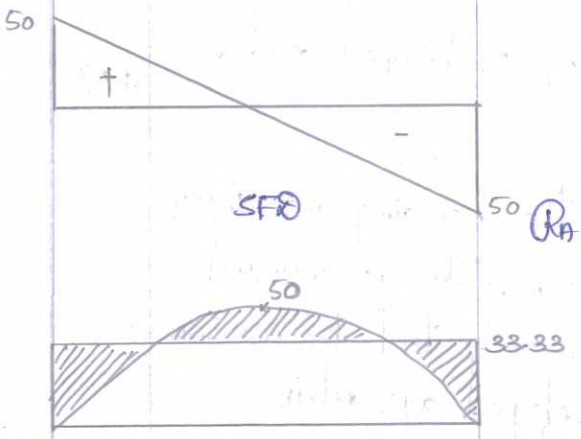
Redundant frames : $m > 2j - 3$

3 x 2 06

3) 25 kN/m



01



$$M_A = M_B = \frac{wl^2}{12}$$

$$= \frac{25 \times 4^2}{12}$$

$$= 33.33 \text{ kN-m}$$

02

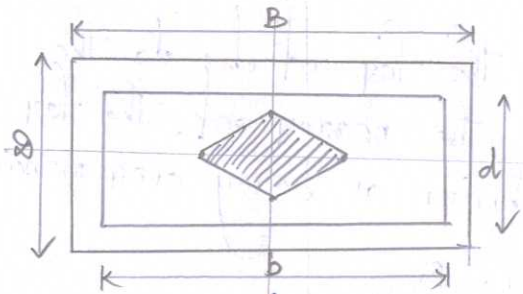
$$R_A = R_B = \frac{25 \times 4}{2}$$

$$= 50 \text{ kN}$$

SFD-01

BMD-02 06

4)



Area of C.S = $BD - bd$

$$I_{yy} = \frac{(BD^3 - db^3) \times 2}{12 \times B} = \frac{2BD^3 - db^3}{6B}$$

02

For no tension in the base, $e = Z/A$

$$= \frac{Z_{yy}}{(BD - bd)}$$

$$e = \frac{DB^3 - db^3}{6B(DB - db)}$$

For no tension on either side of YY axis $= 2e$

$$= \frac{DB^3 - db^3}{3B(DB - db)}$$ 02

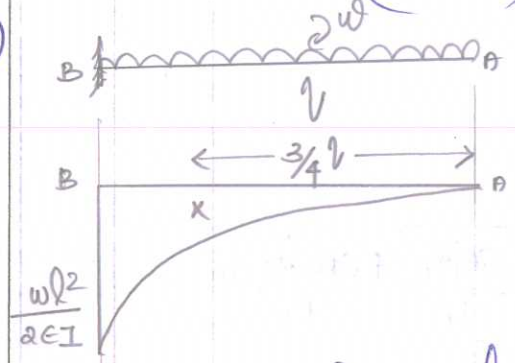
Similarly eccentricity w.r.t XX axis

$$e = \frac{Z_{xx}}{A} = \frac{BD^3 - bd^3}{6D(DB - db)}$$

For no tension on either side of XX axis $= 2e$

$$= \frac{BD^3 - bd^3}{6D(DB - db)}$$ 02 06

5)



02

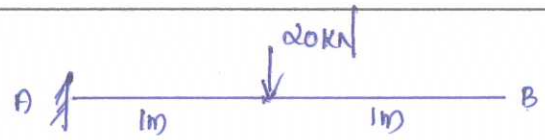
θ_{AB} = Area of M/EI diagram

$$= 1/3 \times l \times \frac{wl^2}{2EI} = \frac{wl^3}{6EI}$$ 02

y_{BA} = Area of M/EI diagram $\times \bar{x}$

$$= \frac{wl^3}{6EI} \times 3/4\, l = \frac{wl^4}{8EI}$$ 02 06

6)



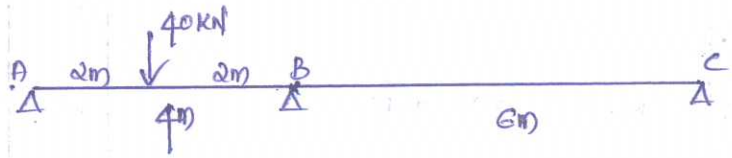
ok

$$\theta_B = \frac{Wl^2}{2EI} = \frac{20\times1^2}{2EI} = \frac{10}{EI}$$ 02

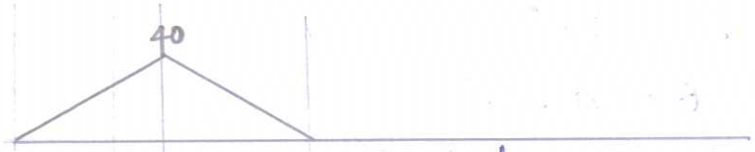
$$y_B = \frac{Wl^3}{3EI} + \frac{Wl^2}{2EI} \times 1m$$

$$= \frac{20\times1^3}{3EI} + \frac{20\times1^2}{2EI}\times1 = \frac{16.67}{EI}$$ 03 06

7)



01



Free moment dgm

$A_1 = \frac{1}{2}\times4\times40$ $\quad A_2 = 0$

$= 80\ kNm^2$ $\quad \bar{x}_2 = 0$

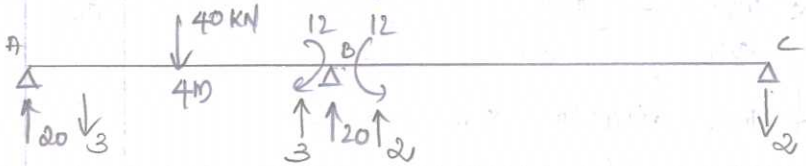
$\bar{x}_1 = 2m$ 01

Apply three moment theorem

$$M_A L_1 + 2M_B(L_1+L_2) + M_C L_2 + \frac{6A_1\bar{x}_1}{L_1} + \frac{6A_2\bar{x}_2}{L_2} = 0$$

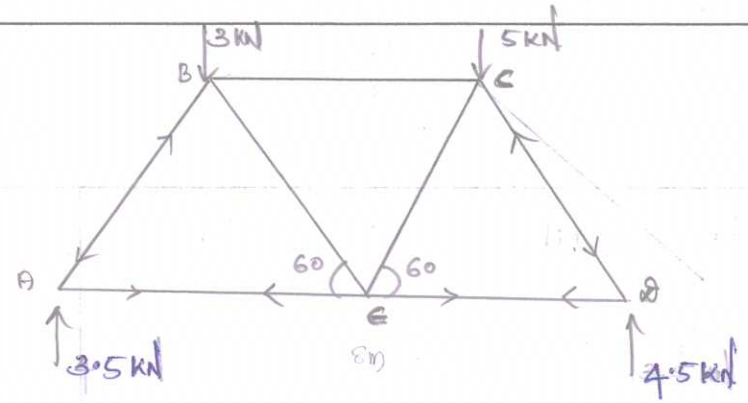
$$2M_B \times 10 = \frac{-6\times80\times2}{4}$$

$$M_B = -12\ kN\text{-}m.$$



02

	$R_A = 17\ kN$ $R_B = 25\ kN$ $R_C = -2\ kN$	02	06	42
	PART-C			
III a)	Given, $d = 12m$			
	$A = \pi/4\, d^2 = 113.09\ mm^2$			
	$I = \pi d^4/64 = 1017.87\ mm^3$			
	$k = \sqrt{I/A} = 3mm$	02		
	Case 1: $7.6\times10^3 = \dfrac{f_c \cdot A}{1+\alpha\left(\frac{500}{3}\right)^2}$			
	Case 2: $22\times10^3 = \dfrac{f_c \cdot A}{1+\alpha\left(\frac{200}{3}\right)^2}$			
	$22\times10^3\left(1+\alpha\left(\frac{200}{3}\right)^2\right) = 7.6\times10^3\left[1+\alpha\left(\frac{500}{3}\right)^2\right]$			
	On solving $\alpha = 1/7901.23$	03		
	Substitute in Case 1			
	$7.6\times10^3 = \dfrac{f_c \times 113.09}{1+\frac{1}{7901.33}\times\left(\frac{500}{3}\right)^2}$			
	$f_c = 303.46\ MPa.$	02	07	
III b)	Calculate Reactions			
	$R_A + R_B = 8\ kN$			
	$\Sigma M_A = 0$			
	$R_B \times 8 + 5\times6 + 3\times2 = 0$			
	$R_B = 4.5\ kN$ $R_A = 3.50\ kN$.	02		



Joint A

$F_{AB} = \frac{3.5}{\sin 60} = 4.04$ kN (C)

$F_{AE} = 4.04 \cos 60 = 2.02$ kN (T) 01

Joint D

$F_{DC} = 4.5/\sin 60 = 5.196$ (C)

$F_{DE} = 5.196 \cos 60 = 2.59$ kN (T) 01

Joint B

$\Sigma V = 0$

$3 + F_{AB} \sin 60 + F_{BE} \sin 60 = 0$

$3 - 3.5 + F_{BE} \times 0.866 = 0$

$F_{BE} = 0.577$ kN (T)

$\Sigma H = 0$

$F_{BC} + 0.577 \times 0.5 - (-4.0415) \times 0.5) = 0$

$F_{BC} = -2.309$ kN (C) 02

Joint E

$\Sigma V = 0$

$F_{EB} \cos 60 + F_{EC} \cos 60 = 0$

$0.577 \cdot \cos 60 + F_{EC} \cos 60 = 0$

$F_{EC} = -0.577$ KN (C)

$\Sigma H = 0$

$F_{ED} - F_{EA} - F_{EB}\cos 60 - F_{EC}\cos 60 = 0$

$F_{ED} = 2.02 + 0.577 \cos 60 + 0.577 \cos 60$

$= 2.597$ KN (T)

Member	Force (KN)
AB	-4.04
BC	-2.309
CD	-5.196
DE	2.598
AE	2.02
BE	0.577
CE	-0.577

02 08

15

IVa) $P = 1000$ KN

$Le = 6m$

$d_o = 300mm$, $d_i = d$

$\alpha = 1/1600$ $f_c = 80$ mPa.

$A = \pi/4 \times (300^2 - d^2)$

$I = \pi/64 (300^4 - d^4)$

02

$$k = \sqrt{I/A}$$

$$= \sqrt{\frac{\pi}{64}\frac{(300^4 - d^4)\times 4}{\pi(300^2 - d^2)}}$$

$$= \sqrt{\frac{300^2 + d^2}{16}}$$ 02

$$P_{cr} = \frac{f_c \cdot A}{1 + \alpha\left(\frac{Le}{k}\right)^2}$$ 01

$$10^6 = \frac{80 \times (70685.83 - 0.785d^2)}{1 + \frac{1}{1600} \times \frac{6000^2}{300^2 + d^2} \times 16}$$

$$(9\times10^{10}) + 10^6 d^2 + (3.6\times10^{11}) = (300^2 + d^2)(5.65\times10^6 - 62.8d^2)$$

$$-62.8d^4 - 10^6 d^2 + 5.8\times10^{10} = 0$$

Let $x = d^2$

$$-62.8x^2 - 10^6 x + 5.8\times10^{10} = 0$$

$x = 23454$, $d = 153.14$ mm

$t = 73.42$ mm. 02 07

IVb) Joint D

$F_{DC} = 10/\sin 30 = 20$ kN (C)

$F_{DE} = 20\cos 30 = 17.32$ kN (T) 02

Joint E

$F_{EC} = 12$ kN (C)

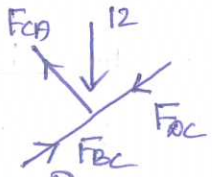
$F_{EA} = F_{ED} = 17.32$ kN (T) — 02

Joint C

$\Sigma V = 0$

$F_{CA} \sin 30 + F_{BC} \sin 30 - F_{CD} \sin 30 - 12 = 0$

$F_{CA} + F_{BC} = 44$



$\Sigma H = 0$

$F_{CA} \cos 30 - F_{BC} \cos 30 + F_{CD} \cos 30 = 0$

$F_{CA} - F_{BC} = -20$

On solving $F_{CA} = 12$ kN (T) $F_{BC} = 32$ kN (C) — 03

Member	Force (kN)
AE	-17.32
ED	-17.32
CD	20 (C)
AC	-12
BC	32
EC	12

01 | 08 | 15

V a) $P_1 = 90$ N $P_2 = 20$ N $P = 110$ N.

$A = 2 \times 2 = 4\ m^2$

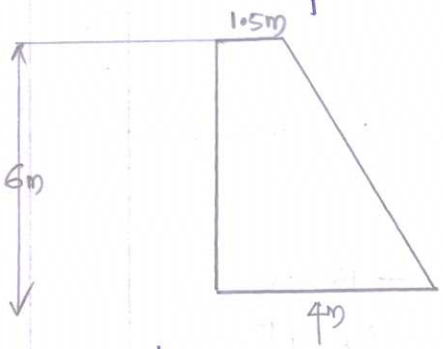
$\bar{y} = 2^3/12 = 1.33$ m. — 02

$$\frac{P}{A} + \frac{P_e \cdot e}{Z} = 2\left(\frac{P}{A} - \frac{P_e \cdot e}{Z}\right)$$ 02

$$\frac{110}{4} + \frac{20e}{1.33} = 2\left(\frac{110}{4} - \frac{20e}{1.33}\right)$$

$e = 0.914\,m$. 02 06

V b)



$a = 1.5m \quad b = 4m \quad H = 6m$

$w = 20\,kN/m^3 \quad \gamma_w = 10\,kN/m^3$ 01

$\bar{x} = \frac{a^2 + ab + b^2}{3(a+b)} = 1.46\,m.$ 01

(a) Reservoir full condition.

$P = \frac{\gamma H^2}{2} = \frac{10 \times 6^2}{2} = 180\,kN$ 01

$W = \frac{1}{2} H(a+b)\,w = \frac{6}{2} \times 5.5 \times 20 = 330\,kN$ 01

$z = \bar{x} + \frac{P}{W} \cdot \frac{H}{3}$

$= 1.46 + \frac{180}{330} \times \frac{6}{3} = 2.56\,m$ 01

$e = z - b/2 = 0.56\,m$

$f_{max}, f_{min} = \frac{W}{b}\left(1 \pm \frac{6e}{b}\right) = 151.87\,kN/m^2,\ 13.125\,kN/m^2$ 01

(a) Reservoir empty condition

$P = 0$

$\Sigma = \bar{x}$

$e = \bar{x} - b/2 = 1.46 - 2 = -0.53$ m. — 02

f_{max} @ heel, f_{min} @ toe $= \frac{W}{b}\left(1 \pm \frac{6e}{b}\right)$

$= \frac{330}{4}\left(1 \pm \frac{6 \times 0.53}{4}\right)$

$= 148.13$ KN/m^2

16.57 KN/m^2 — 01 | 09 | 15

VIa) $P = 100$ kN

$e = 20$ mm, $f_{max} = 43.5$ MPa

$\Sigma = \frac{\pi d^3}{32}$ — 02

$43.5 = \frac{P}{A} + \frac{Pe}{\Sigma}$

$43.5 = \frac{10^5}{(\pi/4 \cdot d^2)} + \frac{10^5 \times 20}{\left(\frac{\pi d^3}{32}\right)}$ — 02

$43.5\, d^3 = 1.27 \times 10^5\, d + 2.03 \times 10^7$

$d = 90.019$ mm. — 02 | 06

VIb) $a = 1.5$m, $b = 4.5$m, $H = 9$m

$\gamma = 18$ KN/m^3, $w = 22$ KN/m^3, $\mu = 0.6$, $f_{max} = 300$ KN/m^2

$k_a = \frac{1 - \sin\phi}{1 + \sin\phi} = 0.22$

$$P = \frac{\gamma H^2}{2} \cdot K_a$$

$$= \frac{18 \times 9^2}{2} \times 0.22 = 158.1 \text{ KN} \quad 01$$

$$W = \tfrac{1}{2} \times (a+b) \times w$$

$$= \tfrac{1}{2} \times 9 \times (1.5+4.5) \times 22 = 594 \text{ KN} \quad 01$$

$$\bar{x} = \frac{a^2+ab+b^2}{3(a+b)} = 1.625 \text{ m} \quad 01$$

$$\Sigma = \bar{x} + \frac{P}{W} \cdot \frac{H}{3}$$

$$= 1.625 + \frac{158.1}{594} \times \frac{9}{3} = 2.424 \text{ m} \quad 01$$

$e = 3 - b/2 = 0.174$ m, $b/6 = 0.75$ m $\quad 01$

$e < b/6$, no tension developed

Check for stability

(i) $e < b/6$ no tension. $\quad 01$

(ii) Overturning

$$\frac{M_o}{M_R} = \frac{P \cdot H/3}{W \cdot \bar{x}} = 0.49 < 1, \text{ safe} \quad 01$$

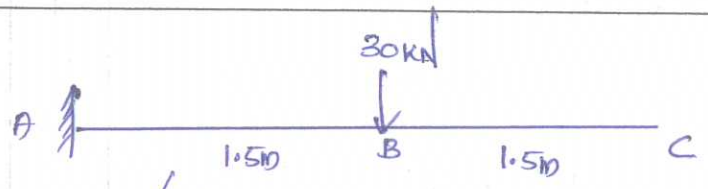
(iii) Sliding

$$\frac{P}{\mu W} = \frac{158.1}{0.6 \times 594} = 0.44, \text{ safe} \quad 01$$

(iv) $f_{max} = \frac{W}{b}\left(1 + \frac{6e}{b}\right) = 162.62 \text{ KN/m}^2 < 300 \text{ KN/m}^2$

safe against bearing pressure. $\quad 01 \quad 09 \quad 15$

VII a)



$E = 200\ kN/mm^2$ $\quad I = 120 \times 10^6\ mm^4$ — 01

(i) Slope & deflection at 1.5 m from fixed end.

$$\theta_B = \frac{WL^2}{2EI} = \frac{30 \times (1500)^2}{2 \times 200 \times 120 \times 10^6} = 1.406 \times 10^{-3}\ rad$$ — 1.5

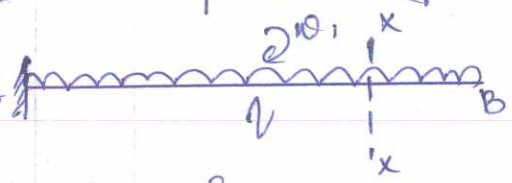
$$y_B = \frac{WL^3}{3EI} = \frac{30 \times 1500^3}{3 \times 200 \times 120 \times 10^6} = 1.41\ mm.$$ — 1.5

(ii) Slope and deflection at free end.

$\theta_C = 1.406 \times 10^{-3}\ rad$ — 1.5

$y_C = 1.41\ mm + (1.406 \times 10^{-3} \times 1500) = 3.52\ mm.$ — 1.5 — 07

VII b)



01

$$M_x = \frac{-wx^2}{2}$$

$$EI \cdot \frac{d^2y}{dx^2} = \frac{-wx^2}{2}$$ — 01

Integrating

$$EI \cdot \frac{dy}{dx} = \frac{-wx^3}{6} + C_1$$

@ $x = l$, $\frac{dy}{dx} = 0$

$$C_1 = \frac{wl^3}{6}$$ — 0.1

$$EI\frac{dy}{dx} = \frac{-wx^3}{6} + \frac{wl^3}{6}$$

$$= \frac{-w}{6}(l^3 - x^3) \quad 01$$

Integrating again

$$EI.y = \frac{w}{6}\left(l^3x - \frac{x^4}{4}\right) + C_2$$

@ $x = l$, $y = 0$

$$0 = \frac{w}{6}\left(l^4 - \frac{l^4}{4}\right) + C_2$$

$$C_2 = \frac{-wl^4}{8} \quad 01$$

$$EI.y = \frac{w}{6}\left(l^3x - \frac{x^4}{4}\right) - \frac{wl^4}{8}$$

Slope, $\frac{dy}{dx} = \frac{-w}{6EI}(l^3 - x^3)$ 01

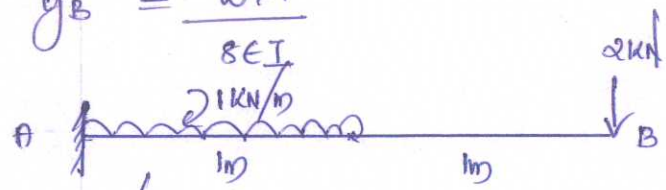
Deflection, $y = \frac{1}{EI}\left\{\frac{w}{6}\left(l^3x - \frac{x^4}{4}\right) - \frac{wl^4}{8}\right\}$ 01

Slope and deflection @ free end.

$$\left(\frac{dy}{dx}\right)_B = \frac{wl^3}{6EI} \quad 01$$

$$y_B = \frac{wl^4}{8EI} \quad 01 \quad 08 \quad 15$$

VIII a)



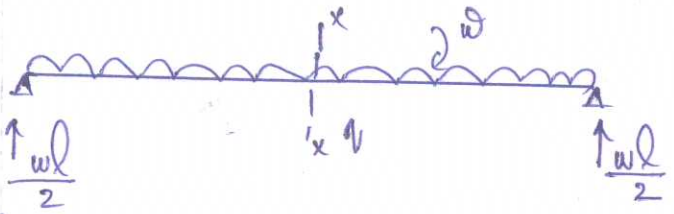
$E = 20\ KN/mm^2 \qquad I = 66.67 \times 10^6\ mm^4$ 01

$$y_B = \frac{w(1000)^4}{8EI} + \frac{w \times (1000)^3}{6EI} \times (1000) + \frac{W \times (2000)^3}{3EI} \quad 02$$

$$= \left(\frac{1 \times 1000^4}{8} + \frac{1 \times (1000)^3 \times 1000}{6} + \frac{(2000)^3 \times 2}{3} \right) \times \frac{1}{EI}$$

$= 222.73$ mm. 03 06

VIII b)



01

$$M_x = \frac{wlx}{2} - \frac{wx^2}{2}$$

$$EI \cdot \frac{d^2y}{dx^2} = \frac{wlx}{2} - \frac{wx^2}{2}$$ 01

Integrating twice

$$EI \cdot \frac{dy}{dx} = \frac{wlx^2}{4} - \frac{wx^3}{6} + C_1$$

$$EI \cdot y = \frac{wlx^3}{12} - \frac{wx^4}{24} + C_1 x + C_2$$ 01

@ $x=0$, $y=0 \Rightarrow C_2 = 0$

@ $x=l$, $y=0$

$$0 = \frac{wl^4}{12} - \frac{wl^4}{24} + C_1 l$$

$$C_1 = \frac{-wl^3}{24}$$ 01

Max slope @ $x=0$

$$EI \cdot \frac{dy}{dx} = \frac{wlx^2}{4} - \frac{wx^3}{6} + C_1$$

$$\frac{dy}{dx} = \frac{-wl^3}{24EI} = \theta_{max}$$ 02

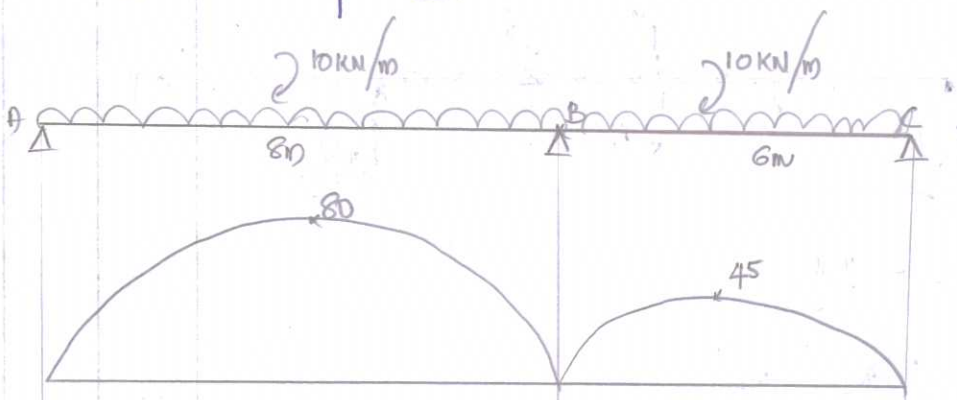
y_{max} @ $x = l/2$

$$EI.y = \frac{wlx^3}{12} - \frac{wx^4}{24} - \frac{wl^3x}{24}$$

$$EI.y = \frac{wl}{12} \times \frac{l^3}{8} - \frac{w}{24} \times \frac{l^4}{16} - \frac{wl^3}{24} \cdot \frac{l}{2}$$

$$y = -5/384 \frac{wl^4}{EI}.$$ 02 08 15

1Xa)



$A_1 = \frac{2}{3} \times 8 \times 80 = 426.67$ KN-m^2 $A_2 = \frac{2}{3} \times 45 \times 6 = 180$ KN-m^2

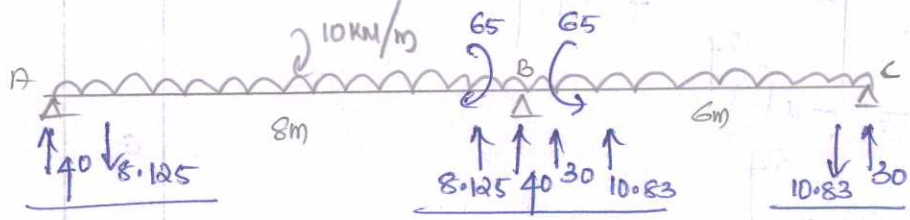
$\bar{x}_1 = 4$m $\bar{x}_2 = 3$m 01

Apply three moment theorem

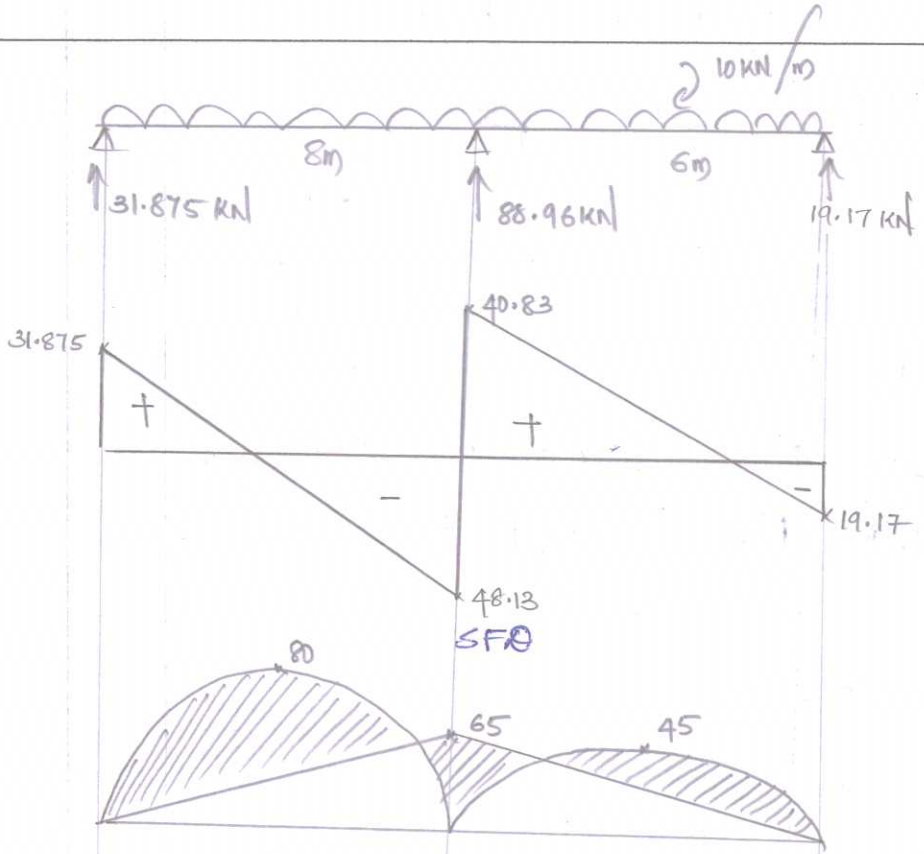
$$M_A l_1 + 2M_B(l_1 + l_2) + M_C l_2 + \frac{6A_1\bar{x}_1}{l_1} + \frac{6A_2\bar{x}_2}{l_2} = 0$$

$$2M_B \times 14 + \frac{6 \times 426.67 \times 4}{8} + \frac{6 \times 180 \times 3}{6} = 0$$

$M_B = 65$ KN-m. 02



$R_A = 31.875$ KN $R_B = 88.96$ KN $R_C = 19.17$ KN 02



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IX b) Fixed End moments

$$M_{FBC} = \frac{-wl^2}{12} = -60 \text{ KN-m}$$

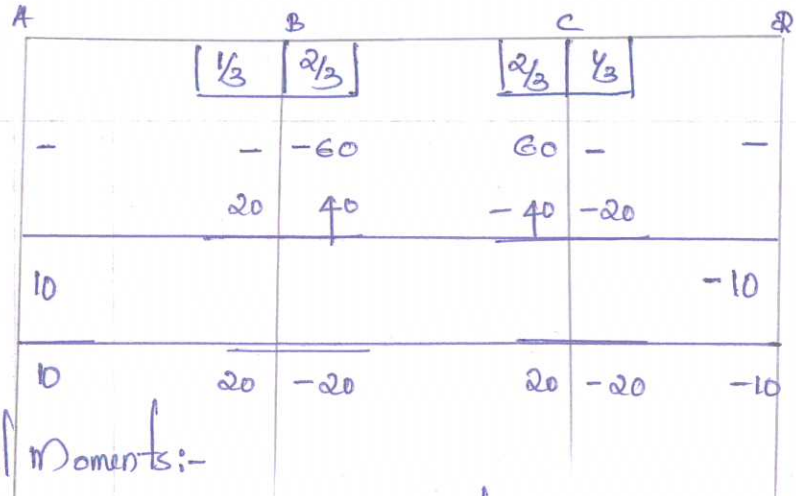
$$M_{FCB} = 60 \text{ KN-m}.$$

02

Distribution Factors

Joint	Member	k	Σk	DF
B	BA	I/4	$\frac{3}{4}$ I	1/3
	BC	3I/6		2/3
C	CB	3I/6	$\frac{3}{4}$ I	2/3
	CD	I/4		1/3

02

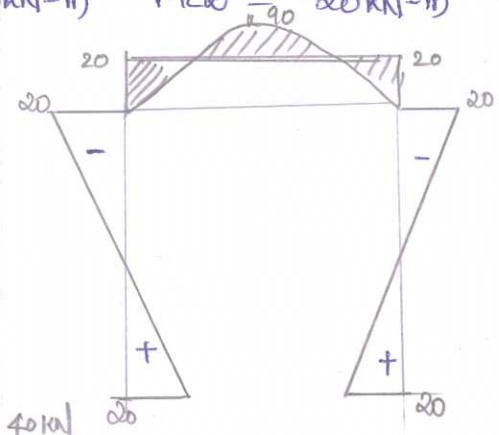


02

Final Moments:-

$M_{AB} = 10\,KN\text{-}m$ $M_{BA} = 20\,KN\text{-}m$ $M_{BC} = -20\,KN\text{-}m$

$M_{CB} = 20\,KN\text{-}m$ $M_{CD} = -20\,KN\text{-}m$ $M_{DC} = -10\,KN\text{-}m$ 01



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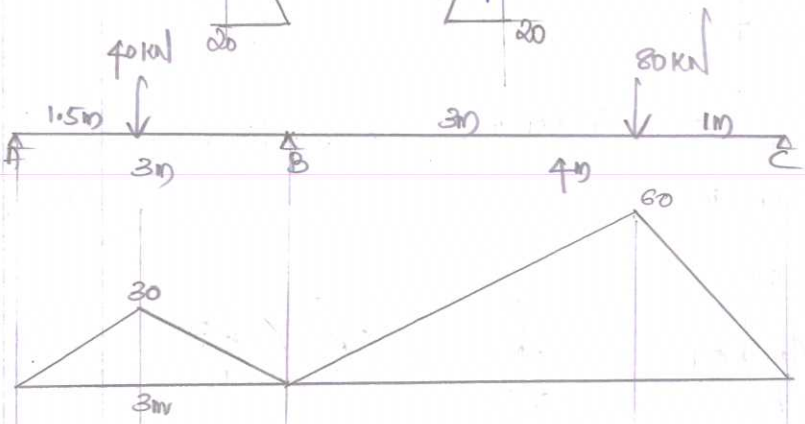
Xa)



$A_1 = \frac{1}{2} \times 3 \times 30 = 45\,KN\text{-}m^2$ $A_2 = \frac{1}{2} \times 4 \times 80 = 160\,KN\text{-}m^2$

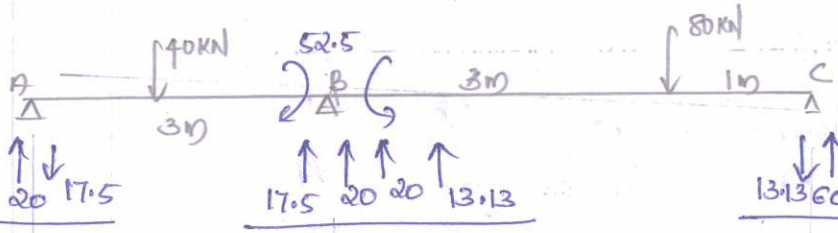
$\overline{x_1} = 1.5m$ $\overline{x_2} = \frac{4+1}{2} = 2.5m.$ 01

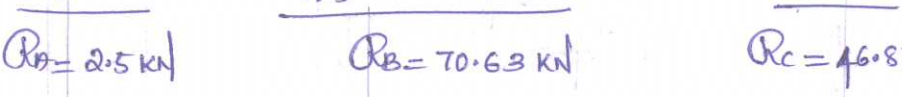
Apply three moment theorem.

$$M_A \cdot l_1 + 2M_B(l_1+l_2) + M_C \cdot l_2 + \frac{6A_1\bar{x}_1}{l_1} + \frac{6A_2\bar{x}_2}{l_2} = 0$$

$$2M_B \times 7 + \frac{6 \times 45 \times 1.5}{3} + \frac{6 \times 160 \times 2.5}{4} = 0$$

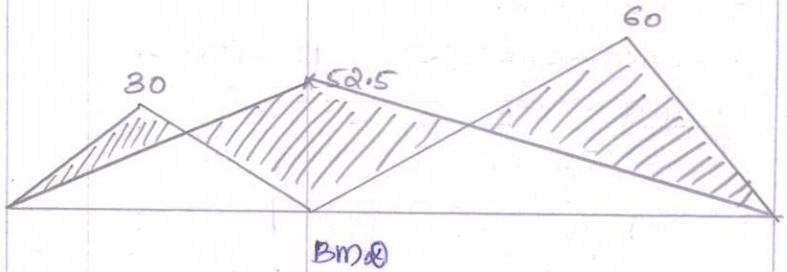
$$M_B = 52.5 \text{ KN-m.}$$ 02





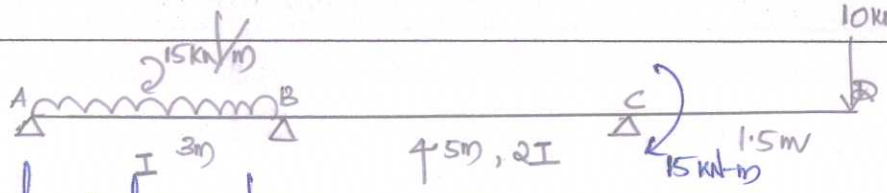
$R_A = 2.5$ KN $R_B = 70.63$ KN $R_C = 46.87$ 02

SFD 01



BMD 01

X.b)



Find End moments :-

$$M_{FAB} = \frac{-wl^2}{12} = \frac{-15 \times 3^2}{12} = -11.25 \text{ KN-m}$$

$M_{FBA} = 11.25$ KN-m

$M_{FBC} = M_{FCB} = 0$

Moment @ joint C $= -10 \times 1.5 = -15$ KN-m. 02

Distribution Factor.

Joint	Members	k	Σk	DF
B	BA	$3/4\ I/3$	$\frac{7I}{12}$	3/7
	BC	$3/4\ 2I/4.5$		4/7

02

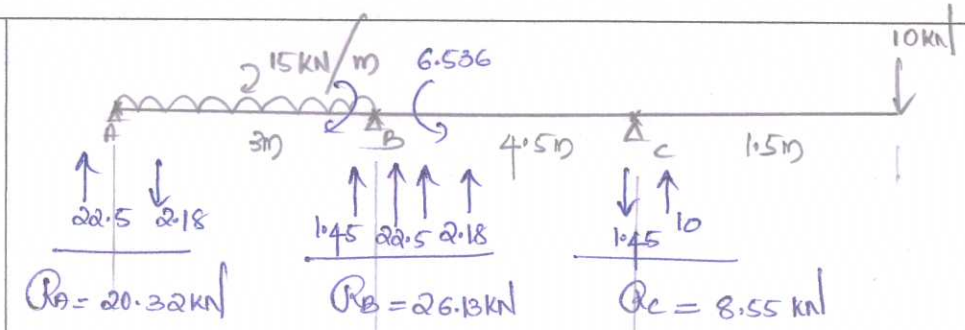
Moment Distribution Table.

A	B		C	
	3/7	4/7		
-11.25	11.25	0	0	-15
11.25	5.625	7.25	15	
0	16.875	7.25	15	-15
	-9.96	-13.28		
0	6.915	-6.03	15	-15
	-0.379	-0.505		
0	6.536	-6.536	15	-15

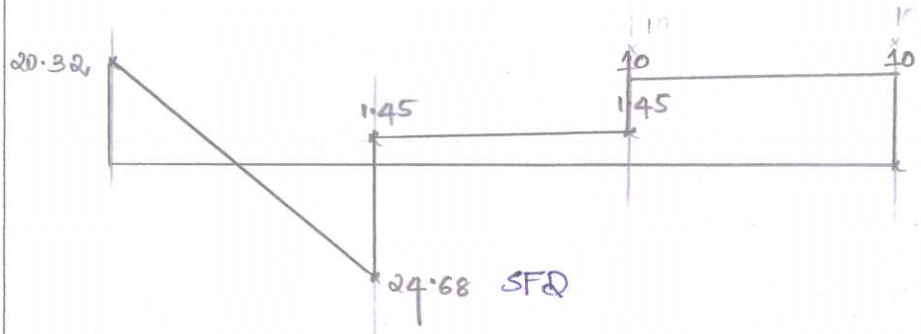
Final moments.

$M_{AB} = 0$ $M_{BA} = 6.536$ KN-m $M_{BC} = -6.536$ KN-m.

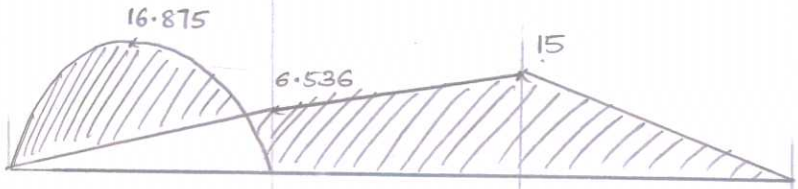
$M_{CB} = -15$ KN-m 02



$R_A = 20.32\ kN$ $R_B = 26.13\ kN$ $R_C = 8.55\ kN$



SFD



BMD.

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x x