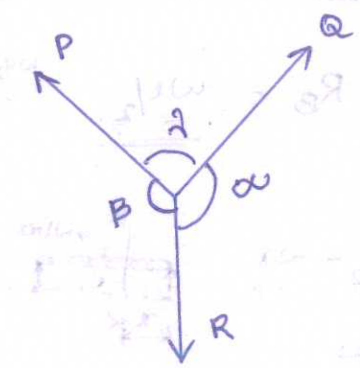
349 D Page-18 April-20..

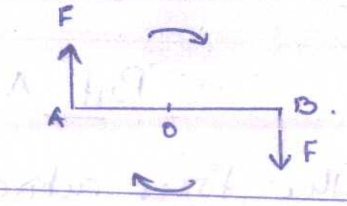
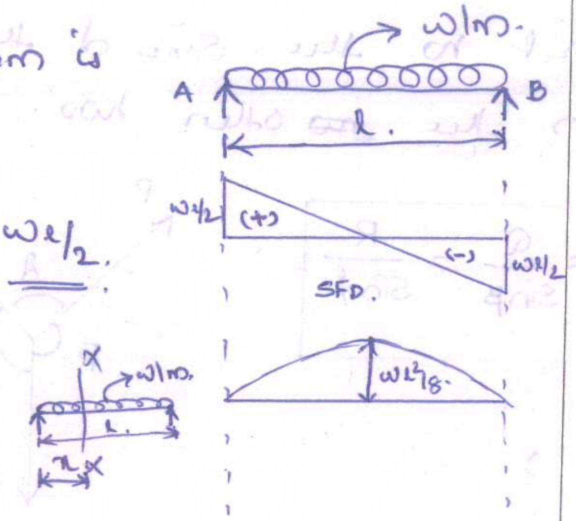
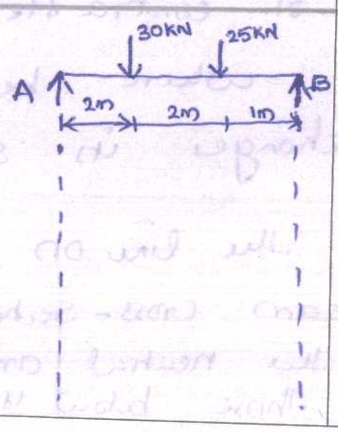
Revision : 2015

COURSE CODE : 3013

Course title : THEROY OF STRUCTURES-I

Qst. No	Scoring indicators	Split Up Marks	Sub Total	Total
	Part - A.			
I	1. If the Forces acting on a particle keep in equilibrium then each Force is proportional to the Sine of the angle between the ~~two~~ other two. $\boxed{\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\gamma}}$ 	2		10
	2. With in the elastic limit the stress is directly proportional to strain. Stress ∝ strain. $\boxed{\sigma = E \cdot e.}$	2		
	3. In a bending beam, a point is known as a point of contra flexure if it is a location where bending moment is zero (changes its sign).	2		
	4. It is the line on the neutral plane on the beam cross-section. All Fibres situated above the neutral axis are under compression and those below it are under tension.	2		

The line or axis of stress is zero. that line is called neutral axis.

	5. Two equal, unlike, parallel forces whose lines of action are different form a couple. 		2	
	Part B			
II	1. As the beam is symmetric.  $R_A = R_B = wl/2$. SFD $SF_{xx} = wl/2 - wx$ $SF_A = wl/2$. $SF_B = -wl/2$. BMD $M_{xx} = \frac{wl}{2}.x - \frac{wx^2}{2}$ $M_A = 0.$, $M_B = \frac{wl^2}{2} - \frac{wl^2}{2} = 0.$ Moment at centre (max. BM.) $M_{max} = \frac{wl^2}{8}.$		6	
	2. $R_A + R_B = 55KN$ $M_A \Rightarrow R_B \times 5 = (25\times4) + (30\times2)$ $5R_B = 100 + 60.$ $R_B = 160/5 = 32KN$ $R_A = 55 - 32 = 23KN$ 		6	

3) $t = T_2 - T_1 = 80^\circ c$

$L = 3m$
$T_1 = 10^\circ c$
$T_2 = 90^\circ c$
$E = 2 \times 10^5 \ N/mm^2$
$\alpha = 12 \times 10^{-6} /^\circ c$
$\beta = 1mm.$

Stress

$$\sigma = \left(\frac{\alpha t L - \beta}{L} \right) \times E$$

6

$$= \left(\frac{12 \times 10^{-6} \times 80 \times 3000 - 1}{3000} \right) \times 2 \times 10^5$$

$$= \underline{125.33 \ N/mm^2}$$

---

4)

$E = 1.2 \times 10^5 \ N/mm^2$
$N = 4.8 \times 10^4 \ N/mm^2$

$$E = 2N(1 + \mu)$$

$$\mu = \frac{E}{2N} - 1 = \frac{1.2 \times 10^5}{2 \times 4.8 \times 10^4} - 1$$

$$\mu = \underline{0.25}$$

6

$$E = 3K(1 - 2\mu)$$

$$K = \frac{E}{3(1-2\mu)} = \frac{1.2 \times 10^5}{3(1 - 2 \times 0.25)}$$

$$= \underline{8 \times 10^4 \ N/mm^2}$$

---

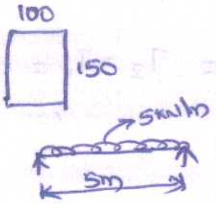
5)
- The material of the shaft is uniform through out
- The twist along the shaft is uniform.
- The shaft is of uniform cross-section throughout
- Cross section of the shaft which are plane before twist remain plane after twist. 6
- All radii which are straight before twist remain straight after twist.

2

6) $w = 5\ kN/m = 5\ N/mm$

$L = 5m$

$b = 100\ mm \quad d = 150\ mm.$



$M = \frac{wl^2}{8} = \underline{15.625\ kNm} = \underline{15.625 \times 10^6\ kN\,mm}$ 6

$I = \frac{bd^3}{12} = \frac{100 \times 150^3}{12} = \underline{42187500\ mm^4}$

$y_{max} = d/2 = 150/2 = 75\ mm.$

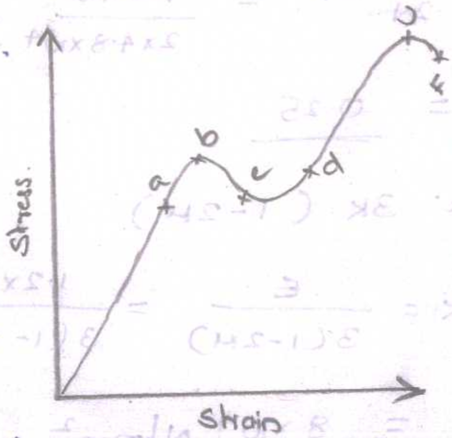
From Bending eqn.

$\frac{M}{I} = \frac{\sigma_{max}}{y_{max}} \Rightarrow \sigma_{max} = \frac{M \times y_{max}}{I}$

$= \frac{15.625 \times 75 \times 10^6}{42187500} = \underline{27.77\ N/mm^2}$

30

7)
a → limit of proportionality.
b → elastic limit
c → upper yield point.
d → lower yield point.
U → ultimate stress/load
F → Fracture stress/load.



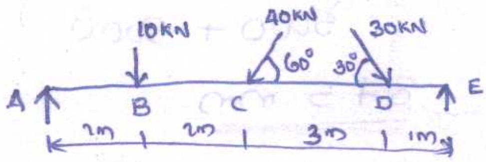
- Limit of proportionality :- It is a load upto which stress strain relation is linear and we get straight line on graph. 6
- Elastic limit (b) :- It is the load upto which material retains elasticity. It may regain its orginal size if load is removed. It is the point beyond which some permanent deformation may occur.
- Upper yield point (c) :- It is the point at which material gives and yield stress is the lowest stress at which material is appreciably deformed without of load.

- lower yield point :- It is the point From which material enters ductile.
- ultimate Stress (U) :- It is the stress develop at maximum load. in brittle materials it corresponds to stress at breaking point.
- Fracture load (F) :- It is the load at which the material get Fractured the stage during U and F the specimen enters a Fracture zone and neck Formation takes place.

Part - C

III a) • $40 \sin 60^\circ = 34.64\,kN$.

• $30 \sin 30^\circ = 15\,kN$



$\Sigma V = 0$

$$R_A + R_E = 10 + 34.64 + 15$$

$$R_A + R_E = \underline{59.64\ kN}$$

$\Sigma M_A = 0$

$$(10 \times 2) + (34.64 \times 4) + (15 \times 7) - R_B \times 8 = 0.$$ 6

$$8R_B = 20 + 69.26 + 105 \Rightarrow 8R_B = 194.26$$

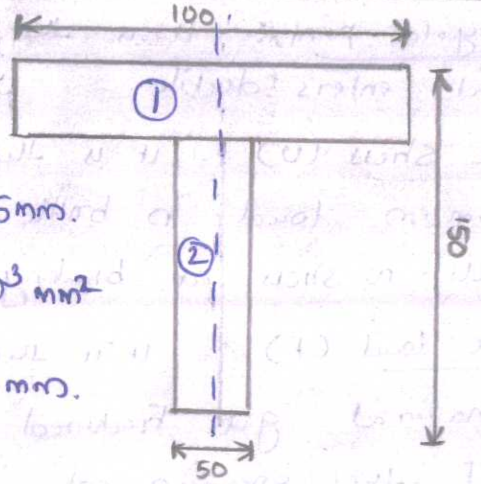
$$R_E = 194.26 / 8 = \underline{24.2825\ kN}.$$

$$R_A = \cancel{24.2825}\ 59.64 - R_B$$

$$= 59.64 - 24.2825$$

$$= \underline{\underline{35.3575\ kN}}$$

b)



$a_1 = 100 \times 50 = 5 \times 10^3 \, mm^2$

$y_1 = 100 + 50/2 = 125 \, mm.$

$a_2 = 100 \times 50 = 5 \times 10^3 \, mm^2$

$y_2 = 100/2 = 50 \, mm.$

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2}.$$ | 9 | 15

$$= \frac{(5000 \times 125) + (5000 \times 50)}{5000 + 5000}$$

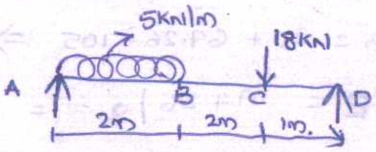
$$= \underline{87.5 \, mm}$$

The Section is Symmetrical about yy axis hence.

$\bar{x} = 100/2 = \underline{50 mm}$.

IV a) $R_A + R_D = (5 \times 2) + 18.$

$R_A + R_D = 10 + 18 = 28.$



$\Sigma M_A = 0.$

$R_D \times 5 = (5 \times 2 \times 2/2) + (18 \times 4)$

$5R_D = 10 + 72$

$R_D = 82/5 = \underline{16.4 \, KN}$ | 6

$R_A = 28 - R_D$

$= 28 - 16.4 = \underline{11.6 \, KN}$

b) $a_1 = 100 \times 20 = 2000 \text{ mm}^2$

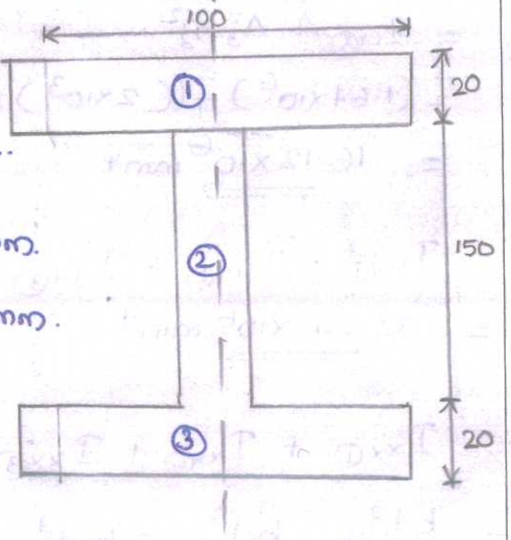
$a_2 = 150 \times 20 = 3000 \text{ mm}^2$

$a_3 = 100 \times 20 = 2000 \text{ mm}^2$.

$y_1 = 170 + 20/2 = 180 \text{mm}$.

$y_2 = 20 + 150/2 = 95 \text{ mm}$.

$y_3 = 20/2 = 10 \text{mm}$



$$\bar{y} = \frac{a_1y_1 + a_2y_2 + a_3y_3}{a_1 + a_2 + a_3}$$

$$= \frac{(2000 \times 180) + (3000 \times 95) + (2000 \times 10)}{2000 + 3000 + 2000}$$

$$= \underline{\underline{95 \text{mm}}}$$

The Section is symmetrical about y axis. 9 15

$$\bar{x} = 100/2 = \underline{\underline{50 \text{mm}}}.$$

Moment of Inertia about YY - axis.

$$I_{G①} = \frac{db^3}{12} = \frac{20 \times 100^3}{12} = \underline{166666.67 \text{ mm}^4}.$$

$$I_{G②} = \frac{db^3}{12} = \frac{150 \times 20^3}{12} = \underline{\underline{100 \times 10^3 \text{ mm}^4}}.$$

$$I_{G③} = \frac{db^3}{12} = \frac{20 \times 100^3}{12} = \underline{\underline{1.67 \times 10^6 \text{ mm}^4}}$$

$$I_{YY①} = I_{G①} + A_1 h_1^2$$
$$= (1.67 \times 10^6) + ((2 \times 10^3) \times (180 - 95)^2)$$
$$= \underline{\underline{16.12 \times 10^6 \text{mm}^4}}.$$

$$I_{YY②} = I_{G②} + A_2 h_2^2$$
$$= (1 \times 10^6) + ((3 \times 10^3) \times (95 - 95)^2)$$
$$= \underline{\underline{1 \times 10^6 \text{ mm}^4}}$$

4

$$I_{yy③} = I_{G③} + A_3 h_3^2$$
$$= (1.67\times10^6) + ((2\times10^3)\times(95-20)^2)$$
$$= \underline{\underline{16.12\times10^6 \text{ mm}^4}}$$

$$I_{yy} = I_{yy①} + I_{yy②} + I_{yy③}$$
$$= \underline{\underline{33.24\times10^6 \text{ mm}^4}}$$

$$I_{xx} = I_{xx①} + I_{xx②} + I_{xx③}$$
$$= \frac{b_1 d_1^3}{12} + \frac{b_2 d_2^3}{12} + \frac{b_3 d_3^3}{12}$$
$$= \left(\frac{100\times20^3}{12}\right) + \left(\frac{40\times150^3}{12}\right) + \left(\frac{100\times20^3}{12}\right)$$
$$= \underline{\underline{5.7583\times10^6 \text{ mm}^4}}$$

V a) i) $*U = \dfrac{\sigma^2}{2E}\times V$

$P = 200\text{ kN} = 200\times10^3\text{ N}$
$b = 500\text{ mm}$
$d = 200\text{ mm}$
$L = 2\text{ m} = 2000\text{ mm}$

$$\sigma = \frac{P}{A} = \frac{200\times10^3}{(500\times200)} = 2\text{ N/mm}^2$$

$$= \frac{2^2}{2\times200\times10^3}\times(500\times200\times2000)$$
$$= \underline{\underline{2000\text{ Nmm}}}$$

*ii) $U^* = \dfrac{\sigma^{*2}}{2E}\times V$
$$= \frac{200^2}{(2\times10^3)\times2}\times(500\times200\times2000)$$
$$= 20\times10^6\text{ Nmm}$$

iii) Modulus of Resilience $= \dfrac{\sigma^{*2}}{2E}$
$$= \frac{200^2}{2\times200\times10^3}$$
$$= \underline{\underline{0.1\text{ N/mm}^2}}$$

9

b) Mechanical Properties of metals.

1) Stiffness (K) :- Ability of a meterial to resist deformation when it is stressed or it can also be defined as load required to create unit deformation in any structure

2) Elasticity :- Property of a material to regain its orginal shape after deformation when external Force is withdrawn.

3) Plasticity :- Property by which material retains the deformation after the withdrawal of load

4) Ductility :- Property by which a metail can be drawn into wires

5) Malleability :- Property by which a material can be drawn into thin sheet.

6) Brittleness :- the property by which material breaks with little permanent deformation.

7) Toughness :- Property of a material to resist fracture due to impact loads.

8) Hardness :- Property by which a material resist Penitration or abrasion.

	6	15

VI. a)

i) Resilience :- Strain energy stored in a member when strained within elastic limits is known as Resilience.

$$\boxed{U = \frac{\sigma^2}{2E} \times V}$$

9

ii) Proof Resilience :- The maximum strain energy stored in a member when strained within elastic limits is known as Proof Resilience.

$$\boxed{U_p = \frac{\sigma^{*2}}{2E} \times V}$$

	iii) Modulus of Resilience:- It is defined as the proof resilience of a material per unit volume. modulus of Resilience = $\frac{\text{Proof Resilience}}{\text{Volume}}$ $= \frac{\sigma^{*2}}{2E}$			
b.	E = 200 GPa = 200×10^9 N/m². L = 2.5 m. A = 1000 mm² = 10^{-3} m² P = 30 KN = 30×10^3 N $V = \pi r^2 h$ $= (10^{-3}) \times 2.5$ $= 2.5 \times 10^{-3}$ $\sigma = 2P/A$ $= \frac{2 \times 30 \times 10^3}{10^{-3}}$ $= 60 \times 10^6$ N/m² $U = \frac{\sigma^2}{2E} \times V$ $= \frac{(60 \times 10^6)^2}{2 \times (200 \times 10^9)} \times (2.5 \times 10^{-3})$ $= 22.5$ Nm		6	15

VII a) $\Sigma V = 0$

$R_A + R_B = 2 + (1 \times 3)$

$R_A + R_B = 5$ — ①.

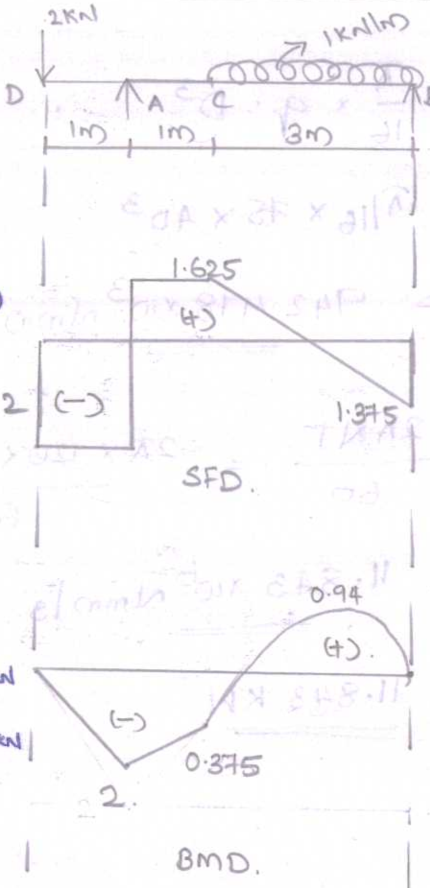
$\Sigma M_B = 0$

$4R_A = (1 \times 3 \times 1.5) + (2 \times 5)$

$R_A = 14.5 / 4$

$= \underline{3.625 \text{ kN}}$

$R_B = \underline{1.375 \text{ kN}}$



SFD

$SF_D = -2 \text{ kN}$, $SF_A = 1.625 \text{ kN}$

$SF_C = 1.625 \text{ kN}$, $SF_B = -1.375 \text{ kN}$

BMD

$M_D = 0$, $M_A = -2 \text{ kNm}$

$M_C = -0.375 \text{ kNm}$, $M_B = 0$.

$M_{max} = 0.94 \text{ kNm}$

10

Point of contraflexure

when $x = 0$.

$M_{xx} = 0.375 + 1.625x - x^2/2 = 0$.

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

$a \rightarrow 0.5x^2$

$b \rightarrow 1.625x$

$c \rightarrow -0.375$

$\underline{x = 3 \text{ m}}$, $\underline{x = 0.25 \text{ m}}$

6/15

b.

$$T = \frac{\pi}{16} \times q \cdot D^3.$$

$D = 40\,mm$
$N = 120\,rpm$
$q = 75\,N/mm^2$

$$= \pi/16 \times 75 \times 40^3$$

$$= \underline{\underline{942.478 \times 10^3}}\ Nmm$$

5 15

$$P = \frac{2\pi NT}{60} = \frac{2\pi \times 120 \times 942.478 \times 10^3}{60}$$

$$\underline{\underline{11.843 \times 10^6}}\ Nmm/s.$$

$$= \underline{\underline{11.843\ KW}}.$$

VIII a) $\Sigma V = 0.$

$R_A + R_B = 4 + 10 + 1$

$R_A + R_B = 15$

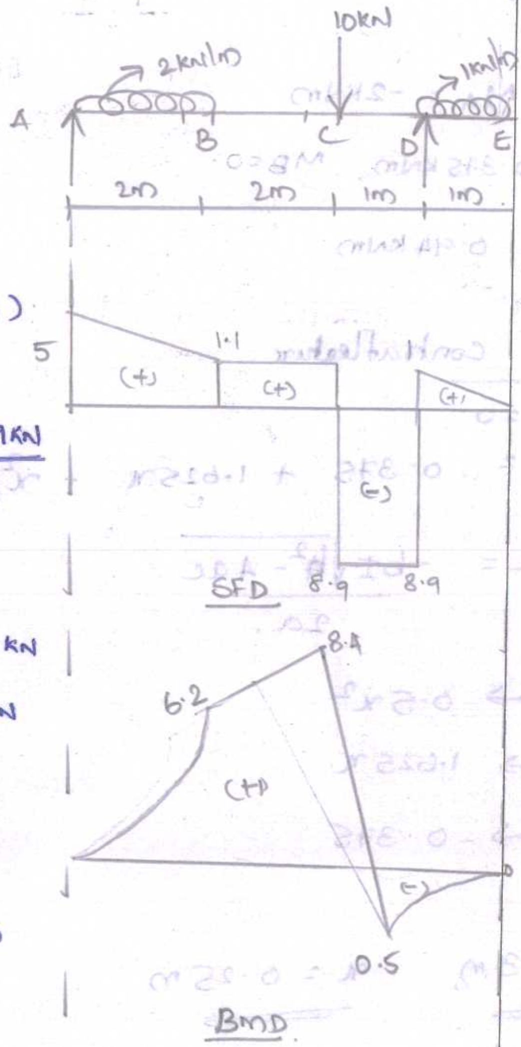
$\Sigma M_A = 0 \Rightarrow 5R_D =$

$((2\times2)\times1 + (10\times4) + (1\times1)\times5.5)$

$5R_D = 49.5$

$R_D = 49.5/5 = \underline{9.9\,KN}$

$R_A = 5.1\ KN$



10

SFD

$SF_A = 5.1\,KN$, $SF_B = 1.1\ KN$

$SF_C = 1.1\ KN$, $SF_D = 1\ KN$

$SF_E = 0$

BMD

$M_A = 0$, $M_B = 6.2\,KNm.$

$M_C = 8.4\,KNm$ $M_D = -0.5\,KNm$

$M_E = 0$

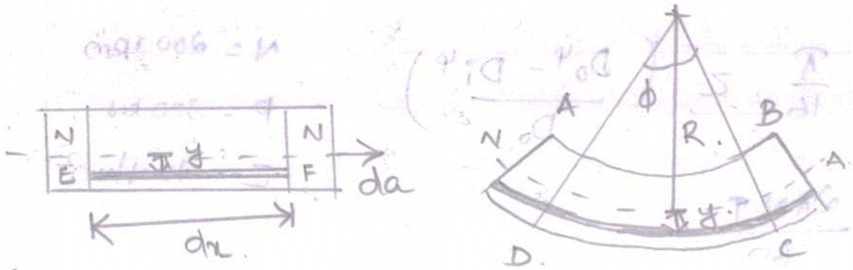
Point of contraflexure

$$M_{xx} = 8.4 - 8.9x = 0.$$

$$\Rightarrow 8.9x = 8.4 = x = \frac{8.4}{8.9} = 0.943m$$

b)	$T = \frac{\pi}{16} \cdot \tau \left(\frac{D_o^4 - D_i^4}{D_o} \right)$ $N = 200\,rpm$ $P = 300\,kN$ $\tau = 70\,N/mm^2$ $P = \frac{2\pi N T}{60}$ $T = \frac{P \times 60}{2\pi N} = \frac{300 \times 10^3 \times 60}{2\pi \times 200}$ $= 14323.945\,Nm$ $\frac{T \times 16}{\pi \times \tau} = \frac{D_o^4 - D_i^4}{D_o}$ $\frac{14.323 \times 10^6 \times 16}{\pi \times 70} = \frac{120^4 - D_i^4}{120}$ ~~$D_i^4 = 120^4 - 1459025566$~~ ~~$D_i^4 = 61454443.39$~~ $120^4 - D_i^4 = 125051040$ $D_i^4 = 120^4 - 125051040$ $= 82308960.01$ $D_i = 95.25\,mm$		5	15

IX a)



Consider a small length dx of beam subjected to simple bending.

R → Radius of Neutral layer.

φ → Angle subtended by AB & CD at centre.

Original length EF = dx.

N'N' = NN = EF = dx.

strain in layers EF = $\frac{\text{changing length}}{\text{original length}}$

$$= \frac{E'F' - EF}{EF}$$

$$= \frac{(R+y)\phi - R\phi}{R\phi}$$

$$= \frac{y}{R} \quad \text{— (1)}$$

9

strain in layers EF = $\frac{\text{Stress}}{E} = \frac{\sigma_b}{E}$ — (2)

From (1) & (2)

$$\frac{\sigma_b}{E} = \frac{y}{R}$$

$$\boxed{\frac{\sigma_b}{y} = \frac{E}{R}} \quad \text{— (3)}$$

Force in layer EF

Force = stress × Area

$= \sigma_b \times da$.

$F = \left(\frac{E}{R} \times y\right) \times da.$

moment about Neutral axis.

Moment = Force x lever arm

$$M = \left(\frac{E}{R} \times y\right) da \times y$$

$$M = \frac{E}{R}\left(y^2 da\right)$$

We know $I = y^2 da$

$$M = \frac{E}{R} \times I$$

$$\boxed{\frac{M}{I} = \frac{E}{R}} \quad \text{--- (4).}$$

From eqns (3) & (4) we can write bending eqn as.

$$\boxed{\frac{M}{I} = \frac{\sigma_b}{y} = \frac{E}{R}}$$

Where,

M → Bending moment

I → Moment of Inertia

σ_b → bending stress.

E → young's modulus.

R → Radius of curvature.

y → Distance of fibre from the Neutral axis.

b) circumferential strain $E = 200$ GPa

$$e_c = \frac{Pd}{2tE}\left(1 - \frac{\mu}{2}\right)$$ $d = 3$m. $t = 20$mm. $P = 3$ N/mm². $\mu = 0.3$

$$= \frac{3 \times 3000}{2 \times 20 \times 200 \times 10^3}\left(1 - \frac{0.3}{2}\right)$$

$$= \underline{9.5625 \times 10^{-4}}$$

$$e_c = \frac{\Delta d}{d}$$

change in dia $= \Delta d = e_c \times d$

$$= 9.5625 \times 10^{-4} \times 3000$$

$$= \underline{\underline{2.868 \text{ mm}}}.$$

6 15

Volumetric strain $e_v = \frac{Pd}{2tE}\left(\frac{5}{2} - 2\mu\right)$

$$= \frac{3 \times 3000}{2 \times 20 \times 200 \times 10^3} \times \left(\frac{5}{2} - 2 \times 0.3\right)$$

$$= \underline{\underline{2.1375 \times 10^{-3}}}.$$

$V = \frac{4}{3}\pi r^3$.

$= \underline{9424777.96}$

$$e_v = \frac{\Delta V}{V}$$

$$\Delta V = e_v \times V$$

$$= 2.1375 \times 10^{-3} \times 942477.961$$

$$= \underline{\underline{20145.46 \text{ mm}^3}}$$

X a) i) The material of the beam is homogenous & isotropic

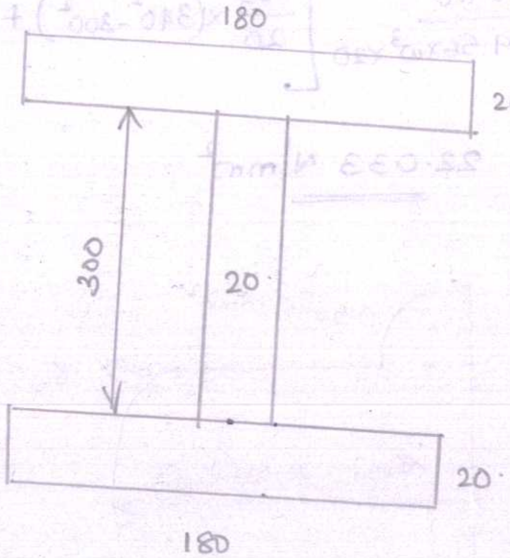
ii) The value of young's modulus of elasticity is the same in tension and compression.

iii) The transverse section which where plane before bending remains plane after bending — 6

iv) The beam is initially straight & longitudinal filaments bend into circular arc with a common centre of curvature.

v) The radius of curvature is large compared with dimensions of the cross-section.

b) — 15



$$I_{xx} = \frac{BD^3}{12} - \frac{bd^3}{12}$$ — 9

$$= \left(\frac{180 \times 340^3}{12}\right) - \left(\frac{160 \times 300^3}{12}\right)$$

$$= \underline{229.56 \times 10^6 \, mm^4}$$

$\frac{9}{21}$

$$Z_1 = \frac{F}{8I}(D^2 - d^2)$$

$$= \frac{40\times10^3}{8\times229.56\times10^6}(340^2 - 300^2)$$

$$= \underline{\underline{0.5575\ N/mm^2}}$$

$$Z_2 = \frac{B}{b}\times\frac{F}{8I}(D^2 - d^2)$$

$$= \frac{180}{20}\times\frac{40\times10^3}{8\times229.56\times10^3}\times(340^2 - 300^2)$$

$$= \underline{\underline{5.018\ N/mm^2}}$$

$$Z_{max} = \frac{F}{Ib}\left[\frac{B}{b}(D^2 - d^2) + \frac{bd^2}{8}\right]$$

$$= \frac{40\times10^3}{229.56\times10^3\times20}\left[\frac{180}{20}\times(340^2 - 300^2) + \frac{20\times300^2}{8}\right]$$

$$= \underline{\underline{22.033\ N/mm^2}}.$$