DIPLOMA EXAMINATION IN ENGINEERING APRIL - 2020

ENGINEERING MATHEMATICS - II

Revision : 2015

Code : 2002 B

Sem : 2

Max. Marks : 75

	PART - A		
I 1.	$(r+1)^{th}$ term $= 15C_2 (3x)^{15-2} \left(\frac{1}{x^2}\right)^2$	1	
	$= 15C_2\, 3^{13} x^{13} \frac{1}{x^4}$	1	2
	$= \underline{15C_2\, 3^{13} x^9}$		
2.	$\lvert A\rvert = 3 - 0 = \underline{3}$		
	$C_{11} = 3, \quad C_{12} = -1 \times 0 = 0$	1	
	$C_{21} = -1 \times 2 = -2, \quad C_{22} = 1 \times 1 = 1$		
	Cofactor $= \begin{bmatrix} 3 & 0 \\ -2 & 1 \end{bmatrix}$ $\quad AdjA = \begin{bmatrix} 3 & -2 \\ 0 & 1 \end{bmatrix}$		2
	$A^{-1} = \dfrac{\begin{bmatrix} 3 & -2 \\ 0 & 1 \end{bmatrix}}{3} = \underline{\begin{bmatrix} 1 & -2/3 \\ 0 & 1/3 \end{bmatrix}}$	1	
3.	$\int (x^6 + x^3)\, dx$	1	2
	$= \frac{x^7}{7} + \frac{x^4}{4} + c$	1	
4.	$A = \int_a^b y\, dx = \int_1^2 (2x-3)\, dx =$	1	
	$= \left[2\frac{x^2}{2} - 3x\right]_1^2 = (x^2 - 3x)_1^2$		2
	$= (4-6) - (1-3) = -2 - -2 = \underline{0}$	1	

5. $\frac{dy}{dx} = e^{x+y}$

$\frac{dy}{e^y} = e^{x} dx$ 1

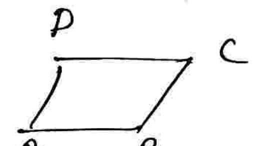
$e^{-y} dy = e^{x} dx$ 1 2

$\frac{e^{-y}}{-1} = e^{x} + c$

$-e^{-y} = e^{x} + c$

PART-B

II 1. Area $= |\vec{AB} \times \vec{AD}|$ 1



$= \begin{vmatrix} i & j & k \\ 1 & -1 & 3 \\ 2 & -7 & 1 \end{vmatrix}$ 2

$= |i(-1+21) - j(1-6) + \hat{k}(-7+2)|$ 2 6

$= |20i + 5j - 5\hat{k}|$ 1

$= \sqrt{20^2 + 5^2 + (-5)^2} = 15\sqrt{2}$

2. Moment about $A = \vec{r} \times \vec{F}$ 1

$\vec{r} = \vec{PA}$ $\vec{F} = i + 2j + k$ 1

$\vec{r} = (2i + 3j + k) - (i + 2j - k)$

$= \hat{i} + \hat{j} + 2\hat{k}$ 1

$\vec{r} \times \vec{F} = \begin{vmatrix} i & j & k \\ 1 & 1 & 2 \\ 1 & 2 & 1 \end{vmatrix}$ 2

$= i(1-4) - j(1-2) + k(2-1)$

$= -3i - \hat{j} + 2k$

Moment $= \sqrt{(-3)^2 + (1)^2 + 1^2} = \underline{\sqrt{11}}$ 1 6

3. $\Delta = \begin{vmatrix} 2 & 3 & 1 \\ 2 & -1 & 4 \\ 3 & 4 & -5 \end{vmatrix}$ 1

$= 2(5-16) - 3(-10-12) + 1(8 - -3)$

$= 2 \times -11 - 3(-22) + 11$

$= -22 + 66 + 11 = \underline{\underline{55}}$

$\Delta_1 = \begin{vmatrix} 11 & 3 & 1 \\ 13 & -1 & 4 \\ 3 & 4 & -5 \end{vmatrix}$

$= 11(5-16) - 3(-65-12) + 1(52+3)$ 1

$= 11 \times -11 - 3 \times -77 + 55$

$= -121 + 231 + 55$

$= \underline{\underline{165}}$

$\Delta_2 = \begin{vmatrix} 2 & 11 & 1 \\ 2 & 13 & 4 \\ 3 & 3 & -5 \end{vmatrix} = 2(-65-12) - 11(-10-12) + 1(6-39)$ 1

$= -154 + 242 - 33$ 6

$= \underline{\underline{55}}$

$\Delta_3 = \begin{vmatrix} 2 & 3 & 11 \\ 2 & -1 & 13 \\ 3 & 4 & 4 \end{vmatrix} = 2(-3-52) - 3(6-39) + 11(8+3)$ 1

$= -110 + 99 + 121$

$= \underline{\underline{110}}$

$x = \frac{\Delta_1}{\Delta} = \frac{165}{55} = \underline{\underline{3}}$ $\quad y = \frac{\Delta_2}{\Delta} = \frac{55}{55} = 1$ 2

$z = \frac{\Delta_3}{\Delta} = \frac{110}{55} = \underline{\underline{2}}$

4. Put $u = \tan^{-1} 5x$. $\therefore$

$$du = \frac{1}{1+25x^2} \cdot 5\,dx. \qquad 1$$

$$\therefore \int_0^{\infty} \frac{(\tan^{-1} 5x)^2}{1+25x^2} 5\,dx = \int_0^{\infty} u^2 \frac{du}{5} \qquad 1$$

$$= \frac{1}{5}\left(\frac{u^3}{3} + c\right)_0^{\infty} \qquad 1$$

$$= \frac{(\tan^{-1} 5x)^3}{3\times 5} + c = \frac{(\tan^{-1} 5x)^3}{15} + c \qquad 1 \qquad 6$$

$$= \frac{(\tan^{-1}\infty)^3}{3\times 5} - \frac{(\tan^{-1} 0)^3}{3\times 5} \qquad 1$$

$$= \frac{\left(\frac{\pi}{2}\right)^3}{15} - 0 = \frac{\pi^3}{120} \qquad 1$$

$=$

5. $$\int x^2 e^{-x}\,dx = x^2 \int e^{-x}\,dx - \int\left(\frac{d}{dx}(x^2)\int e^{-x}\,dx\right)dx \qquad 1$$

$$= x^2 \frac{e^{-x}}{-1} - \int 2x \frac{e^{-x}}{-1}\,dx \qquad 1$$

$$= -x^2 e^{-x} + 2\int x e^{-x}\,dx \quad -(1).$$

Also $$\int x e^{-x}\,dx = x\int e^{-x}\,dx - \int\left(\frac{d}{dx}(x)\int e^{-x}\,dx\right)dx \qquad 1$$

$$= x\frac{e^{-x}}{-1} - \int \frac{e^{-x}}{-1}\,dx \qquad 1$$

$$= -xe^{-x} + \int e^{-x}\,dx$$

	$\int x e^{-x} dx = -x e^{-x} + \int e^{-x} dx$	1
	$= -x e^{-x} + \frac{e^{-x}}{-1}$	
	(1) becomes $\int x^2 e^{-x} dx = -x^2 e^{-x} + 2\left[-x e^{-x} - e^{-x}\right] + c$	1	6
6.	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$	
	$\frac{y^2}{b^2} = 1 - \frac{x^2}{a^2}$	1
	$y^2 = b^2\left(1 - \frac{x^2}{a^2}\right)$.	
	At x-axis $y = 0$. At the point of intersection	
	$b^2\left(1 - \frac{x^2}{a^2}\right) = 0$	
	$a^2 b^2 = x^2 b^2$	1
	$x = \pm a$	
	Volume $V = \pi \int_a^b y^2 dx$	1
	$= \pi \int_{-a}^{a} b^2\left(\frac{a^2 - x^2}{a^2}\right) dx$	
	$= \pi \times \frac{b^2}{a^2} \int_{-a}^{a} (a^2 - x^2) dx$	1
	$= \frac{\pi b^2}{a^2}\left(a^2 x - \frac{x^3}{3}\right)_{-a}^{a}$	

$$V = \frac{\pi b^2}{a^2}\left[\left(a^3 - \frac{a^3}{3}\right) - \left(-a^3 + \frac{a^3}{3}\right)\right]$$ 1

$$= \frac{\pi b^2}{a^2}\left[\frac{4a^3}{3}\right]$$ 1 6

$$= \frac{4}{3}\pi a b^2 \text{ cubic units}$$

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7. $P = 2\cot x$

$$IF = e^{\int P\,dx} = e^{\int 2\cot x\,dx}$$

$$= e^{2\log \sin x} = e^{\log \sin^2 x}$$

$$= \sin^2 x$$

$$\frac{dy}{dx} + 2y\cot x = 3x^2\operatorname{cosec}^2 x \quad \text{—(1)}$$

(1) $\times$ IF $\Rightarrow$

$$\frac{dy}{dx}\sin^2 x + 2y\cot \sin^2 x = 3x^2\operatorname{cosec}^2 x \cdot \sin^2 x$$

$$\frac{d}{dx}(y \cdot \sin^2 x) = 3x^2 \times \frac{1}{\cancel{\sin^2 x}} \cdot \cancel{\sin^2 x}$$

$$\int \frac{dy}{dx}(y \cdot \sin^2 x)\,dx = \int 3x^2\,dx$$

$$y \cdot \sin^2 x = \cancel{3}\frac{x^3}{\cancel{3}} + c$$

$$y \cdot \sin^2 x = x^3 + c$$

PART- C

Unit - I

III 1.	$\vec{a}+\vec{b} = 5\hat{i}-\hat{j}+3\hat{k}+\hat{i}+3\hat{j}-5\hat{k}$ $= 6\hat{i}+2\hat{j}-8\hat{k}$	2	
	$\vec{a}-\vec{b} = 5\hat{i}-\hat{j}-3\hat{k}-(\hat{i}+3\hat{j}-5\hat{k})$ $= 4\hat{i}-4\hat{j}+2\hat{k}$	2	8
	If $(\vec{a}+\vec{b})\cdot(\vec{a}-\vec{b})=0$, the vectors are perpendicular	2	
	$(\vec{a}+\vec{b})\cdot(\vec{a}-\vec{b}) = (6\hat{i}+2\hat{j}-8\hat{k})\cdot(4\hat{i}-4\hat{j}+2\hat{k})$ $= 24-8-16$ $= 0$	2	
2.	Projection vector of a on b is $a\cos\theta$. $a\cos\theta = \frac{\vec{a}\cdot\vec{b}}{b} = \frac{(\hat{i}-\hat{j})(\hat{i}+\hat{j})}{\sqrt{1^2+1^2}}$	4	7
	$= \frac{1\times1 - -1\times1}{\sqrt{2}} = 0.$	3	
IV 1.	$\vec{F} = (5\hat{i}+2\hat{j}+3\hat{k})-(\hat{i}-2\hat{j}+4\hat{k})$ $= 4\hat{i}+4\hat{j}-\hat{k}$	2	
	$\vec{r} = (\hat{i}+-2\hat{j}+4\hat{k})-(-2\hat{i}+3\hat{j}+5\hat{k})$ $= 3\hat{i}-5\hat{j}-\hat{k}$	2	

Vector Moment $= \vec{r} \times \vec{F}$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -5 & -1 \\ 4 & 4 & -1 \end{vmatrix}$$ 2

$= \hat{i}(5+4) - \hat{j}(-3+4) + \hat{k}(12+20)$

$= \underline{\underline{9\hat{i} - \hat{j} + 32\hat{k}}}$ 8

Moment $= \sqrt{9^2 + (-1)^2 + 32^2}$ 2

$= \underline{\underline{\sqrt{1106} \text{ units.}}}$

2. $\left(x^3 - \frac{1}{x^2}\right)^5 =$

$(x^3)^5 - 5C_1 (x^3)^4 \left(\frac{1}{x^2}\right)^1 + 5C_2 (x^3)^3 \left(\frac{1}{x^2}\right)^2 -$

$5C_3 (x^3)^2 \left(\frac{1}{x^2}\right)^3 + 5C_4 (x^3) \left(\frac{1}{x^2}\right)^4 - \left(\frac{1}{x^2}\right)^5$ 2

$= (x^3)^5 - \frac{5x^{12}}{x^2} + \frac{10x^9}{x^4} - \frac{10x^6}{x^6} + \frac{5x^3}{x^8} - \frac{1}{x^{10}}$ 3 7

$= \underline{\underline{x^{15} - 5x^{10} + 10x^5 - 10 + \frac{5}{x^5} - \frac{1}{x^{10}}}}$ 2

Unit II (5)

V

1. $AB = \begin{bmatrix} 1 & 1 & -1 \\ 2 & 0 & 3 \\ 3 & -1 & 2 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 1+0-2 & 0+1-3 \\ 2+0+6 & 0+0+9 \\ 3+0+4 & 0-1+6 \end{bmatrix}$

$= \begin{bmatrix} -1 & -2 \\ 8 & 9 \\ 7 & 5 \end{bmatrix}$ 2

$(AB)C = \begin{bmatrix} -1 & -2 \\ 8 & 9 \\ 7 & 5 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 & -4 \\ 2 & 0 & -2 & -1 \end{bmatrix}$

$= \begin{bmatrix} -1+-4 & -2+0 & -3+4 & 4+2 \\ 8+18 & 16+0 & 24-18 & -32-9 \\ 7+10 & 14+0 & 21-10 & -28-5 \end{bmatrix}$ 2

$\begin{bmatrix} -5 & -2 & 1 & 6 \\ 26 & 16 & 6 & -41 \\ 17 & 14 & 11 & -33 \end{bmatrix}$

$BC = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 & -4 \\ 2 & 0 & -2 & -1 \end{bmatrix}$

$= \begin{bmatrix} 1+0 & 2+0 & 3+0 & -4+0 \\ 0+2 & 0+0 & 0-2 & 0+-1 \\ 2+6 & 4+0 & 6-6 & -8-3 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 & -4 \\ 2 & 0 & -2 & -1 \\ 8 & 4 & 0 & -11 \end{bmatrix}$ 2

$A(BC) = \begin{bmatrix} 1 & 1 & -1 \\ 2 & 0 & 3 \\ 3 & -1 & 2 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 & -4 \\ 2 & 0 & -2 & -1 \\ 8 & 4 & 0 & -11 \end{bmatrix}$

$= \begin{bmatrix} 1+2-8 & 2+0-4 & 3-2+0 & -4-1+11 \\ 2+0+24 & 4+0+12 & 6+0+0 & -8+0-33 \\ 3-2+16 & 6+0+8 & 9+2+0 & -12+1-22 \end{bmatrix}$

$$A(BC) = \begin{bmatrix} -5 & -2 & 1 & 6 \\ 26 & 16 & 6 & -41 \\ 17 & 14 & 11 & -33 \end{bmatrix}$$ 2 8

$$\underline{\underline{A(BC) = (AB)C}}$$

2. $$A^2 = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

$$= \begin{bmatrix} 1+8+21 & 2+10+24 & 3+12+27 \\ 4+20+42 & 8+25+48 & 12+30+54 \\ 7+32+63 & 14+40+72 & 21+48+81 \end{bmatrix}$$

$$\begin{bmatrix} 30 & 36 & 42 \\ 66 & 81 & 96 \\ 102 & 126 & 150 \end{bmatrix}$$ 2

$$A^3 = \begin{bmatrix} 30 & 36 & 42 \\ 66 & 81 & 96 \\ 102 & 126 & 150 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

$$\begin{bmatrix} 30+144+294 & 60+180+336 & 90+216+378 \\ 66+324+672 & 132+405+768 & 198+486+864 \\ 102+504+1050 & 204+630+1200 & 306+756+1350 \end{bmatrix}$$ 3

$$\begin{bmatrix} 468 & 576 & 684 \\ 1062 & 1305 & 1548 \\ 1656 & 2034 & 2412 \end{bmatrix}$$

$$3A^2 = \begin{bmatrix} 90 & 108 & 126 \\ 198 & 243 & 288 \\ 306 & 378 & 480 \end{bmatrix}$$ 1

$A^3 - 3A^2 + 2I =$

$$\begin{bmatrix} 468 & 576 & 684 \\ 1062 & 1305 & 1548 \\ 1656 & 2034 & 2412 \end{bmatrix} - \begin{bmatrix} 90 & 108 & 126 \\ 198 & 243 & 288 \\ 306 & 378 & 450 \end{bmatrix} + \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} \quad 1 \quad 7$$

$$= \begin{bmatrix} 380 & 468 & 558 \\ 864 & 1064 & 1260 \\ 1350 & 1656 & 1964 \end{bmatrix}$$

VI 1. $A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & -2 & -1 \\ 0 & 3 & -5 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 3/2 \\ 9 \end{bmatrix}$ 1

$A^{-1} = \dfrac{\text{adj } A}{|A|} \qquad X = A^{-1}B.$

$|A| = 26 + 5 + 3 = \underline{34}$

$C_{11} = 13, \; C_{12} = 5, \; A_{13} = 3$

$A_{21} = 8 \quad A_{22} = -10, \; A_{23} = -6$ 2

$A_{31} = 1 \quad A_{32} = 3 \quad A_{33} = -5$

Cofactor Matrix $= \begin{bmatrix} 13 & 5 & 3 \\ 8 & -10 & -6 \\ 1 & 3 & -5 \end{bmatrix}$ 1

Adj A $= \begin{bmatrix} 13 & 8 & 1 \\ 5 & -10 & 3 \\ 3 & -6 & -5 \end{bmatrix}$ ~~2~~ 1

$A^{-1} = \dfrac{1}{34} \begin{bmatrix} 13 & 8 & 1 \\ 5 & -10 & 3 \\ 3 & -6 & -5 \end{bmatrix}$ 1

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{34}\begin{bmatrix} 13 & 8 & 1 \\ 5 & -10 & 3 \\ 3 & -6 & -5 \end{bmatrix}\begin{bmatrix} 1 \\ 3/2 \\ 9 \end{bmatrix}$$ 1

$$= \frac{1}{34}\begin{bmatrix} 13 + 12 + 9 \\ 5 - 15 + 27 \\ 3 - 9 - 45 \end{bmatrix}$$

$$= \frac{1}{34}\begin{bmatrix} 34 \\ 17 \\ -51 \end{bmatrix} = \begin{bmatrix} \frac{34}{34} \\ \frac{17}{34} \\ \frac{-51}{34} \end{bmatrix} = \begin{bmatrix} 1 \\ 1/2 \\ -3/2 \end{bmatrix}$$ 1 8

$\underline{\underline{x = 1,\ y = 1/2\ \ z = -3/2}}$

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2. $AB = \begin{bmatrix} 3 & 4 \\ 8 & 5 \end{bmatrix}\begin{bmatrix} 0 & 8 \\ 8 & 5 \end{bmatrix} = \begin{bmatrix} 32 & 44 \\ 40 & 89 \end{bmatrix}$

$|AB| = 2848 - 1760 = 1088$ 1

Cofactor Matrix of $AB = \begin{bmatrix} 89 & -40 \\ -44 & 32 \end{bmatrix}$ 1

$\text{Adj } AB = \begin{bmatrix} 89 & -44 \\ -40 & 32 \end{bmatrix}$ 1

$(AB)^{-1} = \frac{1}{1088}\begin{bmatrix} 89 & -44 \\ -40 & 32 \end{bmatrix}$.

Cofactor Matrix of $A = \begin{bmatrix} 5 & -8 \\ -4 & 3 \end{bmatrix}$, Cofactor $B = \begin{bmatrix} 5 & -8 \\ -8 & 0 \end{bmatrix}$ 1 7

$\text{Adj}A = \begin{bmatrix} 5 & -4 \\ -8 & 3 \end{bmatrix}$, $\text{Adj}B = \begin{bmatrix} 5 & -8 \\ -8 & 0 \end{bmatrix}$ 1

$A^{-1} = -\frac{1}{17}\begin{bmatrix} 5 & -4 \\ -8 & 3 \end{bmatrix}$, $B^{-1} = \frac{1}{-64}\begin{bmatrix} 5 & -8 \\ -8 & 0 \end{bmatrix}$ 1

$B^{-1}A^{-1} = \frac{-1}{17} \times \frac{-1}{64}\begin{bmatrix} 5 & -8 \\ -8 & 0 \end{bmatrix}\begin{bmatrix} 5 & -4 \\ -8 & 3 \end{bmatrix} = \underline{\frac{1}{1088}}\begin{bmatrix} 89 & -44 \\ -40 & 32 \end{bmatrix}$ 1

Unit III

VII

1) a) $\int \frac{x^2}{x^2+1}\,dx = \int \frac{x^2+1-1}{x^2+1}\,dx$ 1

$= \int \frac{x^2+1}{x^2+1}\,dx - \int \frac{1}{x^2+1}\,dx$ 1 4

$= \int 1\,dx - \int \frac{1}{1+x^2}\,dx$ 1

$= x - \tan^{-1}x + c$ 1

2 b) $\int \frac{x}{x+1}\,dx = \int \frac{x+1-1}{x+1}\,dx$ 1

$= \int \left(\frac{x+1}{x+1} - \frac{1}{x+1}\right) dx$ 1

$= \int 1\,dx - \int \frac{1}{x+1}\,dx$ 1 4

$= x - \log|x+1| + c$ 1

2. $\int \frac{e^x(x+1)\,dx}{\sin(xe^x)}\,dx$. — (1)

put $u = xe^x$ 1

$du = xe^x + e^x\,dx$ 2

$= e^x(x+1)\,dx$.

(1) becomes $\int \frac{1}{\sin u}\,du = \int \operatorname{cosec} u\,du$ 1 7

$= \log(\operatorname{cosec} u - \cot u)| + c$ 2

$= \log\left(\operatorname{cosec}(xe^x) - \cot(xe^x)\right) + c$ 1

VIII. 1. $\int \frac{1}{1+\cos x}\, dx$

$\int \frac{1}{1+\cos x} \frac{(1-\cos x)}{(1-\cos x)}\, dx$ 2

$= \int \frac{1-\cos x}{1-\cos^2 x}\, dx = \int \frac{1-\cos x}{\sin^2 x}\, dx$ 2

$= \int \frac{1}{\sin^2 x}\, dx - \int \frac{\cos x}{\sin x} \cdot \frac{1}{\sin x}\, dx.$ 2

8

$= \int \operatorname{cosec}^2 x\, dx - \int \cot x \operatorname{cosec} x\, dx$ 1

$= -\cot x - (-\operatorname{cosec} x) + c$ 1

$= \underline{\underline{-\cot x + \operatorname{cosec} x}} + c$

2. $\int_0^{\pi/2} \sin^3 x\, dx = \int_0^{\pi/2} \frac{3\sin x - \sin 3x}{4}\, dx$ 1

$= \frac{1}{4}\left[-3\cos x + \frac{\cos 3x}{3}\right]_0^{\pi/2}$ 2

$= \frac{1}{4}\left[-3\cos \pi/2 + \frac{\cos 3\pi/2}{3}\right] - \frac{1}{4}\left[-3\cos 0 + \frac{\cos 0}{3}\right]$ 2

$= \frac{1}{4}(0) - \frac{1}{4}\left[-3 + \frac{1}{3}\right]$ 1 7

$= -\frac{1}{4}\left[-\frac{8}{3}\right] = \underline{\underline{\frac{2}{3}}}$ 1

Unit IV

IX

1. $2x+y=1 \Rightarrow y=1-2x.$

Area $= \int_a^b (f(x)-\phi(x))\,dx$ 2

$= \int_a^b 1-2x-(x^2-6x+4)\,dx.$ 1

For finding a & b $f(x)=\phi(x)$

$1-2x = x^2-6x+4$ 1

$x^2-6x+4+2x-1=0$

$x^2-4x+3=0 \Rightarrow (x-3)(x-1)=0$

$x=3$ or 1

$A = \int_1^3 1-2x-(x^2-6x+4)\,dx$ 8

$= \int_1^3 (-x^2+4x-3)\,dx = \left(-\frac{x^3}{3}+4\frac{x^2}{2}-3x\right)_1^3$ 2

$= \left(-\frac{3^3}{3}+2\cdot 3^2-3\times 3\right)-\left(-\frac{1}{3}+2-3\right)$ 2

$= 4/3$ Sq. units

2. $\frac{d^2y}{dx^2} = \sin^2 x + xe^x$

$\frac{d^2y}{dx^2} = \frac{1}{2}(1-\cos 2x)+xe^x$ 1

integrate on both sides

$\int \frac{d^2y}{dx^2} = \frac{1}{2}\int (1-\cos 2x)\,dx + \int xe^x\,dx$ 1

$\frac{dy}{dx} = \frac{1}{2}\int (1-\cos 2x)\,dx + \int xe^x\,dx$ — (1)

$$I_1 = \int x e^x dx = x\int e^x dx - \int 1 e^x dx$$ 2

$$= xe^x - e^x + c_1$$

① becomes

$$\frac{dy}{dx} = \frac{1}{2}\left[x - \frac{\sin 2x}{2}\right] + xe^x - e^x + c_1$$ 1

$$\int \frac{dy}{dx} = \frac{1}{2}\int x\,dx - \frac{1}{4}\int \sin 2x\,dx + \int xe^x - e^x + c_1$$ 1 7

$$y = \frac{x^2}{4} + \frac{\cos 2x}{8} + e^x(x-2) + c_1$$ 1

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x

1. $$\frac{dy}{dx} = \frac{xy^2 + x}{yx^2 + y}$$

$$\frac{dy}{dx} = \frac{x(y^2+1)}{y(x^2+1)}$$ 1

$$dy \cdot y(x^2+1) = dx \cdot x(y^2+1)$$ 1

$$\frac{y\,dy}{y^2+1} = \frac{x\,dx}{x^2+1}$$

$$\int \frac{y}{y^2+1} dy = \int \frac{x}{x^2+1} dx \quad -①$$ 1

put $u = x^2+1 \qquad du = 2x\,dx$ 1

$$x\,dx = \frac{du}{2}$$

$$\int \frac{x}{x^2+1} dx = \int \frac{du}{2u} = \frac{1}{2}\log u$$

$$= \frac{1}{2}\log(x^2+1) + c_1$$ 1

Similarly $\int \frac{y}{y^2+1}\,dy = \frac{1}{2}\log(y^2+1) + c_2$ 1

(1) becomes

$\frac{1}{2}\log(1+y^2) + c_2 = \frac{1}{2}\log(1+x^2) + c_1$ 1

$\frac{1}{2}\left[\log(1+y^2) - \log(1+x^2)\right] = c_1 - c_2 = c$

$\log\left(\frac{1+y^2}{1+x^2}\right) = 2c$ 1 8

$\frac{1+y^2}{1+x^2} = e^{2c}$ ie $(1+y^2) = k(1+x^2)$

when $k = e^{2x}$

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2. $(1+x^2)\frac{dy}{dx} + y = e^{\tan^{-1}x}$

Dividing the eqn by $(1+x^2)$ 1

$\frac{dy}{dx} + \frac{y}{1+x^2} = \frac{e^{\tan^{-1}x}}{1+x^2}$ ...(1) 1

$IF = e^{\int p\,dx} = e^{\int \frac{1}{1+x^2}dx} = e^{\tan^{-1}x}$ 2

(1) $\times\ e^{\tan^{-1}x} \Rightarrow$

$e^{\tan^{-1}x}\cdot\frac{dy}{dx} + e^{\tan^{-1}x}\cdot\frac{y}{1+x^2} = \left(e^{\tan^{-1}x}\right)^2 \frac{1}{1+x^2}\,dx$ 1 7

$\frac{d}{dx}\left(e^{\tan^{-1}x}\cdot y\right) = \int\left(e^{\tan^{-1}x}\right)^2\cdot\frac{1}{1+x^2}\,dx$ 2

$y\cdot e^{\tan^{-1}x} = \frac{\left(e^{\tan^{-1}x}\right)^2}{2} + c$