DIPLOMA EXAMINATION IN ENGINEERING APRIL-2020

ENGINEERING MATHEMATICS-I

Revision : 2015

Sem : 1

Subj. Code : 1002 A

Max.Marks: 75

ANSWER KEY

I

PART-A

1. $\sin\theta = \frac{5}{13}$

$\cos\theta = \sqrt{1-\sin^2\theta}$ 1

$= \sqrt{1-\frac{25}{169}} = \sqrt{\frac{169-25}{169}} = \sqrt{\frac{144}{169}} = \frac{12}{13}$ 1 2

2. $\sin A = \frac{3}{5}$ $\cos A = \sqrt{1-\frac{9}{25}} = \sqrt{\frac{25-9}{25}} = \sqrt{\frac{16}{25}}$

$= \frac{4}{5}$ 1

$\sin 2A = 2\sin A\cos A$

$= 2 \times \frac{3}{5} \times \frac{4}{5} = \frac{24}{25}$ 1 2

3. $C = 180 - (A+B)$

$= 180 - 105 = 75^0$

$\frac{a}{\sin A} = \frac{b}{\sin B}$ 1

$\frac{5}{\sin 45} = \frac{b}{\sin 60^0}$ $b\sin 45 = 5\sin 60$

$b = \frac{5\sin 60}{\sin 45} = \frac{5 \times \frac{\sqrt3}{2}}{1/\sqrt2}$ 2

$\frac{5\sqrt3}{2} \times \sqrt2 = \frac{5\sqrt6}{2} =$ 1

$\frac{5\sqrt3\,\cancel{\sqrt2}}{\sqrt2\,\cancel{\sqrt2}} = \frac{5\sqrt3}{\sqrt2}$

4. $\lim_{x \to 0} \frac{ax+b}{cx+d} = \frac{a \times 0 + b}{c \times 0 + d} = \frac{b}{d}$ ② 2 2

5. $y = 3x^2 + x - 2$

$\frac{dy}{dx} = 6x + 1$ 1

$\left.\frac{dy}{dx}\right|_{(1,2)} = 6 \times 1 + 1$ 1 2

$= 7$

II

PART-B

1. $\sqrt{3}\cos x + \sin x = R\sin(x + \alpha)$

$= R[\sin x \cos\alpha + \cos x \sin\alpha]$ 2

Equating the similar terms 1

$\sqrt{3} = R\sin\alpha$ — ①

$1 = R\cos\alpha$ — ②

Squaring and adding ① & ② 1 6

$4 = R^2$, $R = \pm 2$

① ÷ ② 1

$\frac{\sqrt{3}}{1} = \frac{R\sin\alpha}{R\cos\alpha}$, $\sqrt{3} = \tan\alpha$

$\alpha = 60^\circ$ 1

$\therefore \sqrt{3}\cos x + \sin x = \pm 2\sin(x + 60^\circ)$

2. $\frac{\sec\theta}{\sec\theta + 1} + \frac{\sec\theta}{\sec\theta - 1}$

$\frac{(\sec\theta - 1)\sec\theta + \sec\theta(\sec\theta + 1)}{\sec^2\theta - 1} =$ 1

$\frac{\sec^2\theta - \sec\theta + \sec^2\theta + \sec\theta}{\tan^2\theta}$ 2

$$\frac{2\sec^2\theta}{\tan^2\theta} = \frac{2 \times \frac{1}{\cos^2\theta}}{\frac{\sin^2\theta}{\cos^2\theta}} = \frac{2}{\sin^2\theta}$$ 2

$$= \underline{\underline{2\,\mathrm{cosec}^2\theta}}$$ 1 6

3. $$\sin 10 \sin 50 \sin 70 = \sin 10 \times -\tfrac{1}{2}\left[\cos(120) - \cos(-20)\right)$$

$$= -\frac{1}{2}\sin 10\left[\cos 120 - \cos 20\right]$$

$$= -\frac{1}{2}\sin 10\left[\cos(180-60) - \cos 20\right]$$ 2

$$= -\frac{1}{2}\sin 10\left[-\cos 60 - \cos 20\right]$$

$$= -\frac{1}{2}\sin 10\left[-\frac{1}{2} - \cos 20\right]$$ 2 6

$$= +\frac{1}{4}\sin 10 + \frac{1}{2}\sin 10\cos 20$$

$$= \frac{1}{4}\sin 10 + \frac{1}{2}\,\frac{1}{2}\left(\sin 30 + \sin(-10)\right)$$ 2

$$= \frac{1}{4}\sin 10 + \frac{1}{4}\sin 30 - \frac{1}{4}\sin 10$$

$$= \frac{1}{4}\sin 10 = \frac{1}{4} \times \frac{1}{2} = \underline{\underline{\frac{1}{8}}}$$

4. $$y = x^n$$

$$y + \Delta y = (x + \Delta x)^n$$ 1

$$\Delta y = (x + \Delta x)^n - y$$ 1

$$= (x + \Delta x)^n - x^n$$

$$\frac{\Delta y}{\Delta x} = \frac{(x + \Delta x)^n - x^n}{\Delta x}$$ 1

$$\lim_{\Delta x \to 0}\frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0}\left[\frac{(x + \Delta x)^n - x^n}{\Delta x}\right]$$ 1

	$\lim_{(x+\Delta x)\to x} \frac{(x+\Delta x)^n - x^n}{x+\Delta x - x} = n x^{n-1}$	2	6
5.	$y = x + \frac{1}{x}$		
	$y' = 1 - \frac{1}{x^2}$	1.	
	$y'' = 0 - -\frac{2}{x^3} = \frac{2}{x^3}$	1	
	$x^2y'' + xy' = x^2\left(\frac{2}{x^3}\right) + x\left(1 - \frac{1}{x^2}\right)$	2	6
	$= \frac{2}{x} + x - \frac{2}{x}$	1	
	$= x + \frac{1}{x}$	1	
	$= y$		
6.	$x = 3\cos t + 5\sin t$		
	$V = \frac{dx}{dt} = -3\sin t + 5\cos t$	2	
	$a = \frac{d}{dt}(-3\sin t + 5\cos t)$	2	
	$= -3\cos t - 5\sin t$		
	$= -1(3\cos t + 5\sin t)$	2	6
	$a = -1s$ $\quad$ $a \propto s$		

8 5

7. Volume of Spherical ballon, $V = \frac{4}{3}\pi r^3$ 1

$\frac{dv}{dt} = 25cc/sec$ $r = 15cm$.

$\frac{d}{dt}(v) = \frac{d}{dt}\left(\frac{4}{3}\pi r^3\right)$ 1

$= \frac{4}{3}\pi \; 3r^2 \frac{dr}{dt}$ 1 6

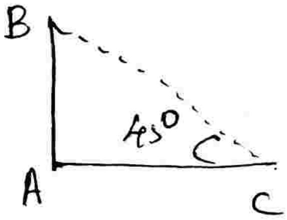
$25 = \frac{4}{3}\pi \times 3 \times 15^2 \left(\frac{dr}{dt}\right)$ 2

$\frac{dr}{dt} = \frac{25 \times 3}{4\pi \times 3 \times 15^2} = \frac{1}{36\pi}$ cm/sec 1

PARTC - C

Unit - 1

III a)



$\cos 45 = \frac{AC}{BC} = \frac{20}{BC}$ 2

ie $\frac{1}{\sqrt{2}} = \frac{20}{BC}$ 2 5

$BC = 20\sqrt{2}$

length of the rope = 28.280m 1

b) $\sin A = \sqrt{1-\cos^2 A} = \sqrt{1-\frac{9}{25}} = \sqrt{\frac{16}{25}} = \frac{4}{5}$ 1

$\sec^2 B = 1+\tan^2 B$

$= 1 + \frac{25}{144} = \frac{169}{144}$

$\sec B = \frac{13}{12} \Rightarrow \cos B = \frac{12}{13}$ 1

	$\sin B = \sqrt{1-\cos^2 B} = \sqrt{1-\frac{144}{169}} = \frac{5}{13}$	1	
	$\sin(A+B) = \sin A\cos B + \cos A\sin B$		
	$= \frac{4}{5}\times\frac{12}{13} + \frac{3}{5}\times\frac{5}{13} = \frac{63}{65}$	1	5
	$\cos(A-B) = \cos A\cos B + \sin A\sin B$		
	$= \frac{3}{5}\times\frac{12}{13} + \frac{4}{5}\times\frac{5}{13} = \frac{56}{65}$	1	
c.	$\sin(A+B)\sin(A-B)$		
	$= (\sin A\cos B + \cos A\sin B)(\sin A\cos B - \cos A\sin B)$	1	
	$(\sin A\cos B)^2 - (\cos A\sin B)^2$	1	
	$= \sin^2 A\cos^2 B - \cos^2 A\sin^2 B$	1	5
	$= \sin^2 A(1-\sin^2 B) - (1-\sin^2 A)\sin^2 B$	1	
	$= \sin^2 A - \cancel{\sin^2 A\sin^2 B} - \sin^2 B + \cancel{\sin^2 A\sin^2 B}$	1	
	$= \sin^2 A - \sin^2 B$		
N a)	$\frac{1+\sin A}{\cos A} = \frac{(1-\sin A)(1+\sin A)}{(1-\sin A)\cos A}$	2	
	$= \frac{1-\sin^2 A}{(1-\sin A)\cos A}$	1	5
	$= \frac{\cos^2 A}{\cos A(1-\sin A)} = \frac{\cos A}{1-\sin A}$	2	

(4) (7)

b) $\cos(90+A) = -\sin A$

$\sec(360+A) = \frac{1}{\cos(360+A)} = \frac{1}{\cos A} = \sec A$ 1

$\tan(180-A) = -\tan A$ 1

$\sec(A-720) = \sec(-[720-A)] = \sec(720-A)$

$= \sec A$ 1

$\sin(540+A) = \sin(360+180+A) = \sin(180+A)$

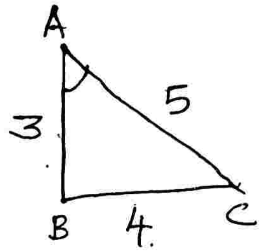
$= -\sin A$ 1 5

$\cot(A-90) = \cot-(90-A) = -\tan A$

$$\frac{\cos(90+A)\sec(360+A)\tan(180-A)}{\sec(A-720)\sin(540+A)\cot(A-90)}$$ 1

$$= \frac{-\sin A \sec A - \tan A}{\sec A \cdot -\sin A - \tan A} = 1$$

$\sin A = \frac{-4}{5}$



$\cos A = \frac{-3}{5}$ 1

$\tan A = \frac{4}{3}$ 1

$\cot A = \frac{3}{4}$ 1 5

$\operatorname{cosec} A = \frac{-5}{4}$ 1

$\sec A = \frac{-5}{3}$ 1

Unit-II

(8)

V.

a) $\cos 4\theta = 1 - 2\sin^2(2\theta)$ 2

$= 1 - 2\sin 2\theta \sin 2\theta$ 1 5

$= 1 - 2.2\sin\theta\cos\theta\, 2\sin\theta\cos\theta$ 1

$= \underline{1 - 8\sin^2\theta\cos^2\theta}$ 1

b) $\dfrac{\sin A + \sin 3A + \sin 5A}{\cos A + \cos 3A + \cos 5A}$

$= \dfrac{\sin 3A + \sin 5A + \sin A}{\cos 3A + \cos 5A + \cos A}$ 1

$= \dfrac{\sin 3A + 2\sin \frac{6A}{2} \cos \frac{4A}{2}}{\cos 3A + 2\cos\frac{6A}{2} \cos\frac{4A}{2}}$ 2 5

$= \dfrac{\sin 3A + 2\sin 3A \cos 2A}{\cos 3A + 2\cos 3A \cos 2A}$ 1

$= \dfrac{\sin 3A(1 + \cancel{2\cos 2A})}{\cos 3A(1 + \cancel{2\cos 2A})} = \underline{\tan 3A}$ 1

C. $A = \cos^{-1}\left[\dfrac{b^2+c^2-a^2}{2bc}\right] = \cos^{-1}\left[\dfrac{5^2+7^2-4^2}{2\times5\times7}\right]$ 1

$= \cos^{-1}\left(\dfrac{58}{70}\right)$

$= \cos^{-1}(0.8286) = \underline{34^\circ 03'}$ 1

$B = \cos^{-1}\left[\dfrac{a^2+c^2-b^2}{2ac}\right] = \cos^{-1}\left(\dfrac{4^2+7^2-5^2}{2\times4\times7}\right)$ 1 5

$= \cos^{-1}\left(\dfrac{40}{56}\right) = \underline{44^\circ 25'}$ 1

$C = 180 - (A+B) = 180 - (78^\circ 28') = \underline{101^\circ 32'}$ 1

(9) (8) (8)

VI

a) $\sin 33 + \cos 63 = \cos 3$

$\sin 33 = \cos 3 - \cos 63$ 1

$= -2\sin\frac{3+63}{2}\sin\frac{3-63}{2}$ 1

$= -2\sin 33 \sin(-30)$ 1 5

$= -2\sin 33 \times -\frac{1}{2}$ 1

$= \underline{\underline{\sin 33}}$ 1

b) RHS. $abc(\cot A + \cot B + \cot C)$

$abc\left[\frac{\cos A}{\sin A} + \frac{\cos B}{\sin B} + \frac{\cos C}{\sin C}\right]$ 1

$= \frac{abc\cos A}{\sin A} + \frac{abc\cos B}{\sin B} + \frac{abc\cos C}{\sin C}$

$= \frac{a}{\sin A} bc\cos A + \frac{b}{\sin B} ac\cos B + \frac{c}{\sin C} ab\cos C$ 1

$= 2R\,bc\cos A + 2R\,ac\cos B + 2R\,ab\cos C$ 1

$= 2R\left[bc\frac{b^2+c^2-a^2}{2bc} + ac\frac{a^2+c^2-b^2}{2ac} + ab\frac{a^2+b^2-c^2}{2ab}\right]$ 1 5

$= 2R\left[\frac{b^2+c^2-a^2+a^2+c^2-b^2+a^2+b^2-c^2}{2}\right]$ 1

$= 2R\left[\frac{a^2+b^2+c^2}{2}\right] = \underline{\underline{R(a^2+b^2+c^2)}}$

C. $A = 35^\circ$, ~~a=7cm~~ $B = 68^\circ$, $C = 25cm$.

$$C = 180 - (A+B)$$

$$= 180 - (35+68) = \underline{\underline{77^\circ}}$$ 1

Also $\frac{a}{\sin A} = \frac{c}{\sin C}$

$$\frac{a}{\sin 35} = \frac{25}{\sin 77}$$ 1

$$\therefore a = \frac{25 \sin 35}{\sin 77} = \underline{\underline{14.71}}$$ 1 5

$$\frac{b}{\sin B} = \frac{c}{\sin C}$$ 1

$$\frac{b}{\sin 68} = \frac{25}{\sin 77}, \quad b = \frac{25 \times \sin 68}{\sin 77} = \underline{\underline{23.78}}$$ 1

Unit-III

VII. a)

$$x = at^2$$

$$\frac{dx}{dt} = 2at$$ 1

$$y = 2at$$

$$\frac{dy}{dt} = 2a.$$ 1 5

$$\frac{dy}{dx} = \frac{(dy/dt)}{\left(\frac{dx}{dt}\right)}$$ 1

2

$$= \frac{2a}{2at} = \underline{\underline{\frac{1}{t}}}$$

(11) -6.

VII, b.

$$x^2+y^2+2gx+2fy+c=0$$

$$\frac{d}{dx}(x^2)+\frac{d}{dx}(y^2)+\frac{d}{dx}(2gx)+\frac{d}{dx}(2fy)+\frac{d}{dx}(c)=0$$ 1

$$2x+2y\frac{dy}{dx}+2g\times 1+2f\frac{dy}{dx}=0$$ 1

$$\frac{dy}{dx}(2y+2f)=-2x-2g$$ 1

$$\frac{dy}{dx}=\frac{-2x-2g}{2y+2f}$$ 1 5

$$=\frac{-2(x+g)}{2(y+f)}=\frac{-(x+g)}{y+f}$$ 1

c.

$$\frac{dy}{dx}=a\cos x-b\sin x$$ 2

$$\frac{d^2y}{dx^2}=-a\sin x-b\cos x.$$ 2 5

$$\frac{d^2y}{dx^2}+y=-a\sin x-b\cos x+a\sin x+b\cos x$$ 1

$$=0$$

VIII, a)

$$\frac{d}{dx}(\cot x)=\frac{d}{dx}\left(\frac{\cos x}{\sin x}\right)$$ 1

$$=\frac{\sin x\frac{d}{dx}(\cos x)-\cos x\frac{d}{dx}(\sin x)}{\sin^2 x}$$ 1

$$=\frac{\sin x\times -\sin x-\cos x.\cos x}{\sin^2 x}$$ 1

(12)

$$\frac{d}{dx}(\cot x) = \frac{-(\sin^2 x + \cos^2 x)}{\sin^2 x} \qquad 1$$

$$= -\frac{1}{\sin^2 x} = -\operatorname{cosec}^2 x \qquad 1 \qquad 5$$

b

$$y = e^x + e^{-x}$$

$$\frac{dy}{dx} = e^x + e^{-x} \times -1 \qquad 1$$

$$= e^x - e^{-x} \qquad 1$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}(e^x - e^{-x}) \qquad 1 \qquad 5$$

$$= e^x - e^{-x} \times -1 \qquad 1$$

$$= e^x + e^{-x}$$

$$= y \qquad 1$$

c.

$$\lim_{x \to 3} \left(\frac{x^3 - 27}{x^2 - 9} \right)$$

$$\lim_{x \to 3} \left[\frac{\frac{x^3 - 27}{x-3}}{\frac{x^2 - 9}{x-3}} \right] = \lim_{x \to 3} \frac{x^3 - 27}{x-3} \div \lim_{x \to 3} \frac{x^2 - 9}{x-3} \qquad 2$$

$$\lim_{x \to 3} \frac{x^3 - 3^3}{x-3} \div \lim_{x \to 3} \frac{x^2 - 3^2}{x-3} \qquad 1 \qquad 5$$

$$3 \times 3^{3-1} \div 2 \times 3^{2-1} \qquad 1$$

$$3 \times 3^2 \div 2 \times 3$$

$$= \frac{27}{6} = \frac{9}{2} \qquad 1$$

IX. a) $y = 3x^2 + x - 2$ at $(1, 2)$

$$\frac{dy}{dx} = 3 \cdot 2x + 1$$

$$= 3 \times 2 \times 1 + 1 = \underline{\underline{7}}$$ 1

Slope of the tangent $= \underline{\underline{7}}$

Eqn of the tangent $y - y_1 = \frac{dy}{dx}(x - x_1)$ 1

$y - 2 = 7(x - 1)$

$y - 2 = 7x - 7$ 1

$\underline{\underline{7x - y - 5 = 0}}$ 5

Eqn of the Normal

$$y - y_1 = \frac{-1}{\left(\frac{dy}{dx}\right)}(x - x_1)$$ 1

$y - 2 = -\frac{1}{7}(x - 1)$ 1

$7(y - 2) = -1(x - 1)$

$7y - 14 = -x + 1$

$\underline{\underline{x + 7y - 15 = 0}}$

IX

b. $y = 4x^3 + 9x^2 - 12x + 2$

$$\frac{dy}{dx} = 12x^2 + 18x - 12$$ 1

$$\frac{dy}{dx} = 0 \Rightarrow 12x^2 + 18x - 12 = 0$$

$$6(2x^2 + 3x - 2) = 0.$$

$$x = \frac{-3 \pm \sqrt{9 - 4 \times 2 \times -2}}{2 \times 2}$$ 1

$$= \frac{-3 \pm \sqrt{9+16}}{4} = \frac{-3 \pm \sqrt{25}}{4}$$

$$= \frac{-3+5}{4}, \frac{-3-5}{4}$$

$$= \frac{2}{4}, -\frac{8}{4}$$

$$= \frac{1}{2}, -2$$

$$\frac{d^2y}{dx^2} = 24x + 18$$ 5

$$\frac{d^2y}{dx^2} \text{ at } x = \frac{1}{2} \quad 24 \times \frac{1}{2} + 18$$ 1

$$= 12 + 18 = 30 > 0$$

$$\frac{d^2y}{dx^2} \text{ at } x = -2 \quad 24 \times -2 + 18$$ 1

$$-48 + 18 = -30 < 0.$$

The function is Maximum at $x = -2$

Max. deflection is $y = 4(-2)^3 + 9(-2)^2 - 12(-2) + 2$ 1

$$= 4 \times -8 + 9 \times 4 + 24 + 2$$

$$= -32 + 36 + 26$$

$$= 4 + 26$$

$$= 30$$

(15) 8

IX c. $S = 2t^3 - 9t^2 + 12t + 6$

$V = \frac{ds}{dt} = 6t^2 - 18t + 12$ 2

$a = \frac{dv}{dt} = 12t - 18$ 1 5

When $a = 0 \Rightarrow 12t - 18 = 0$ 1

$12t = 18$

$t = \frac{18}{12} = \frac{3}{2}$ sec.

X a. Length $= x$, breadth $= y$, perimeter $= P$.

$2x + 2y = P$

$P = 2x + 2y$ 1

$y = \frac{P - 2x}{2}$ — (1)

Area, $A = xy = x\left(\frac{P - 2x}{2}\right) = \frac{1}{2}(Px - 2x^2)$ 1

At. Max. $\frac{dA}{dx} = 0$

$\Rightarrow \frac{1}{2}(P - 4x) = 0$

$P - 4x = 0$, $P = 4x$, $x = P/4$. 1 5

$\therefore y = \frac{P - 2x}{2} = \frac{P - 2 \times \frac{P}{4}}{2} = \frac{P}{4}$

$\frac{d^2A}{dx^2} = \frac{d}{dx}\left(\frac{1}{2}(P - 4x)\right) = \frac{1}{2}(-4) < 0$. 2

Since $\frac{d^2A}{dx^2} < 0$, A is maximum

at $x = \frac{P}{4}$, $y = \frac{P}{4}$

b. $y = x^3 - 2x^2 + x + 1$

$\frac{dy}{dx} = 3x^2 - 4x + 1$ 1

tangent is ||el to x-axis $\therefore \frac{dy}{dx} = 0$ 1

$3x^2 - 4x + 1 = 0$ 1 5

$x = \frac{4 \pm \sqrt{16-12}}{6} = \frac{4 \pm 2}{6}$ 2

$= (1, \frac{1}{3})$

$x = 1 \quad ; \quad x = \frac{1}{3}$

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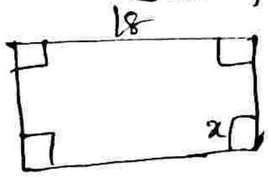
c. Let 'x' be the side of the square sheet which is cut from each of the corners



1

Length of the open box $= 18 - 2x$

breadth " $= 18 - 2x$

height $= x$.

$\therefore$ Volume $= lbh$

$V = (18-2x)(18-2x)x$ 2

$= 4x^3 - 72x^2 + 324x$. 5

$= 12(x^2 - 12x + 27)$.

At maximum

$\frac{dv}{dx} = 0 \Rightarrow 12(x^2 - 12x + 27) = 0$. 1

$(x-9)(x-3) = 0$.

$x = 3$ or $x = 9$ 1

$x = 9$ is inadmissible

$\therefore$ The side of the square sheet cut off $= 3$ cm