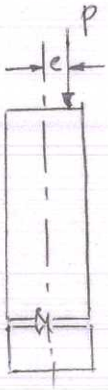
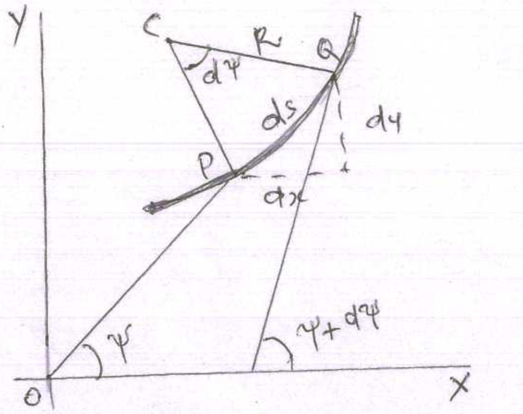
REV. (15) 4014

THEORY OF STRUCTURES - II

Qn.No		Split score	Total score
	PART-A		
I. 1	Slenderness ratio (λ) is the ratio of the effective length of a column (Le) and the least radius of gyration (k) about the axis under consideration. λ greater than 80 long columns and λ less than 80 short columns.	2	
2	A load whose line of action does not coincide with the axis of column.	2	
3	1.To avoid tension in the masonry at the base of the dam 2. To safeguard the dam from overturning 3. To prevent the sliding of dam 4. To prevent the crushing of masonry at the base of the dam. (Any 2x1)	2	
4	Stiffness criterion of the beam design that the beam should be stiff enough to resist the deflection of the beam.	2	
5	<u>Carry over factor:</u> It is the ratio of moment produced at a joint to the moment applied to the other joint without displacement. of the beam.	2	
			10
	<u>PART (B)</u>		
II) 1	<u>SOLUTION</u>		
	Length of column, l = 1.5m Both ends hinged $L_e = l = 1.5m$	1	
	Area of the column, $A = 1590mm^2$ $A = \pi/4 d^2 = 1590mm^2$	1	
	$\therefore$ $d = (1590 x 4/\pi)^{1/2} = 45$ mm		
	Slenderness ratio λ = Le/kmin.	1	
	$K_{min.} = (I_{min.}/A)^{1/2} = ((\pi/64)d^4 / (\pi/4)d^2)^{1/2}$	1	
	$\therefore$ k min. = d/4 = 45/4 = 11.25mm		
		1	
	Slenderness ratio λ = Le/kmin. = 1500/11.25		
	λ = <u>133</u>	1	6

Q. No.	Answer	Marks	Total
2.	Due to eccentric loading a column is subjected to 1. direct load 2. moment due to eccentricity of load	1	
	Consider a column is subjected a compressive load *P* with an eccentricity *e* then: Stress due to direct load = P/A	2	
	Where A is the area of the column Bending stress = M/Z	2	
	Where M= bending moment due to eccentric load P=P x e Z= section modules about the axis of bending (fig) 	1	6
3	Advantages of fixed beam	1.5	
	1.The beam is stiffer ,stronger and more stable	1.5	
	2.The slope at both the ends of the elastic curve is zero	1.5	
	3.Due to fixing moments ,the maximum bending moment at the centre is reduced	1.5	
	4.The deflection of beam at its centre is very much reduced.		6
4	A beam AB is under the action of load.	1(FIG)	
	Let PQ an elementary arc of the curve in which the neutral layer of the beam bends.		
	Let ds= length of the beam PQ R= Radius of the arc From the geometry of the fig, 		
	ds=R. dψ	1	
	R=ds/ dψ=dx/ dψ 1/R= dψ/dx	1	
	X and Y the coordinates of point P tan ψ =dy/dx ...since ψ is very small , tan ψ = ψ	1	
	∴ $d\psi/dx = d^2y/dx^2$ ....(∵ 1/R= dψ/dx)		
	We know M/I = E/R or M = EIx1/R $\underline{M = EI \times d^2y/dx^2}$	2	6

5 Span l = 2m = 2000mm

Load ,w = 5kN/m

EI= 2.5x 10^{12} N-mm^2

Slope iB = wl^3 /6EI

= 5x(2000)^3 /6x(2.5x10^{12}) 2

= <u>0.0027 rad</u> 1

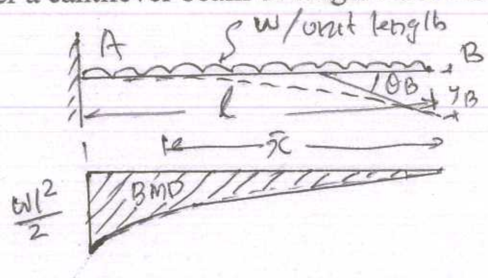
Deflection ,y = = wl^4/8EI 2

= 5x(2000)^4 /8x(2.5x10^{12})

<u>= 4mm</u> 1 6

Consider a cantilever beam of length *l* and carrying a uniformly distributed load *w*

6



Area of BM diagram = $wl^2/2 * (l) *1/3$ = wl^3/6 1

Distance between CG of bending moment diagram and B

$\bar{X}$ =3/4*(l) 1

According to Mohrs theorem 1,change of slope between A and B is

θB = A/EI 1

<u>θB = wl^3 /6EI radians</u> 1

According to Mohrs theorem II, intercepts of tangents on A nad B is

Yb=A$\bar{x}$/EI = wl^3 *(l) *3/4*6EI 1

<u>Yb = wl^4 /8EI</u> 6

7 Stiffness factor is the moment required to rotate the end ,while acting on it through a unit angle with out translation of the other end. 1

2

<u>Stiffness factor for a beam having one end fixed and the other end free</u>

Moment of a beam having one end fixed and the other end free = M = 4EI/l*ϴ

Substituting ϴ = 1 in the above equation

Stiffness factor ,k1= 4EI/l

<u>Stiffness factor for a beam having simply supported ends</u> 2

Moment of a beam having simply supported ends = M = 3EI/l*ϴ

Substituting ϴ = 1 in the above equation

Stiffness factor,k2 = 3EI/l

2 6

<u>PART (C)</u>

III a) <u>Sol.</u>

$L = 6\,m = 6000\,mm$, $d = 100\,mm$, $F.S = 3$ 1

both ends fixed , $L_e = \frac{l}{2} = \frac{6}{2} = 3\,m = 3000\,mm$

$I_{xx} = \frac{bd^3}{12} = \frac{1}{12} \times 200 \times 100^3$ 1

$= 0.1667 \times 10^8\ mm^4$

$I_{yy} = \frac{db^3}{12} = \frac{1}{12} \times 100 \times 200^3$ 1

$= 06667 \times 10^8\ mm^4$ 1

$\therefore I_{min.} = I_{xx} = 0.1667 \times 10^8\ mm^4$ 1

Eulers crippling load $= \frac{\pi^2 EI}{L_e^2}$

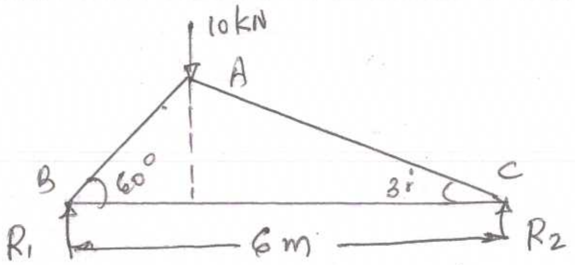
$= \frac{\pi^2 \times 200 \times 0.1667 \times 10^8}{(3000)^2}$ 1

$= 3657\ kN$

Safe load on column $= \frac{3657}{3}$ $\left(\because \frac{P_{cr}}{F.S}\right)$ 2 8

$= \underline{\underline{1219\ kN}}$

II (b)

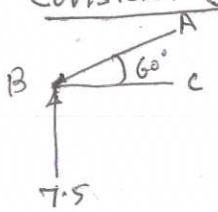


for Reactions R_1 and R_2, taking moments about C

$R_1 \times 6 = 10 \times 4.5$, $R_1 = 7.5$ kN

$R_2 = 10 - 7.5 = 2.5$ kN | 2

Consider joint B



Resolving vertically

$7.5 - F_{AB} \sin 60° = 0$

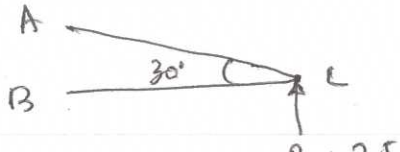
$F_{AB} = \frac{7.5}{\sin 60°} = 8.66$ kN | 2

Resolving horizontally

$F_{AB} \cos 60° = F_{BC}$

$F_{BC} = 8.66 \times \cos 60° = 4.33$ kN | 2

Consider joint C



Resolving vertically

$2.50 - F_{AC} \sin 30° = 0$

$F_{AC} = \frac{2.5}{\sin 30} = 5.00$ kN | 2

Resolving horizontally

$F_{AC} \cos 30° = F_{BC}$

$F_{BC} = 5 \times \cos 30°$

$= 4.33$ kN

Result

Forces on the members | 8

$F_{AB} = 8.66$ kN - Compression

$F_{BC} = 4.33$ kN - Tension

$F_{AC} = 5.00$ kN - compression

IV a) Solution

$D = 38$ mm, $d = 38 - (2 \times 2.5) = 33$ mm

Area, $A = \frac{\pi}{4}(D^2 - d^2) = \frac{\pi}{4}(38^2 - 33^2) = 88.75\pi$ mm^2 | 1

Least radius of gyration, $K = \sqrt{I/A}$ ~~$= \sqrt{14.05 \times}$~~ | 1

$I = \frac{\pi}{64}(D^4 - d^4) = \frac{\pi}{64}(38^4 - 33^4) = 14.05 \times 10^3 \pi$ mm^4 | 1

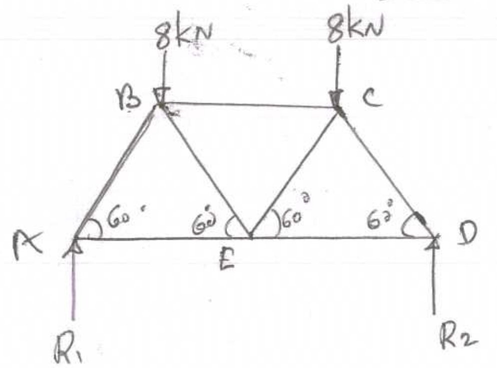
$K = \sqrt{I/A} = \sqrt{\frac{14.05 \times 10^3 \pi}{88.75 \pi}} = 12.6$ mm | 2

∴ Rankines crippling load = $P_R = \dfrac{\sigma_{cs} \cdot A}{1 + a \left(\dfrac{L_e}{k}\right)^2}$ 2

$$P_R = \frac{335 \times 88.75\,\pi}{1 + \frac{1}{7500}\left[\frac{2.3 \times 10^3}{12.6}\right]^2}$$

$P_R = \underline{17.16\ \text{kN}}$ 1 8

IV b)

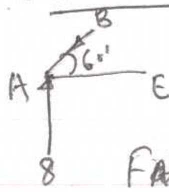


Sol

due to symmetry

$R_1 = R_2 = \dfrac{8+8}{2} = 8$ kN 1

Consider joint A



$\Sigma V = 0$

$F_{AB} \sin 60^\circ - 8 = 0$

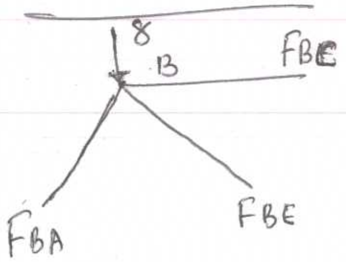
$F_{AB} = 9.24$ kN 1

$\Sigma H = 0$

$F_{AB} \cos 60^\circ - F_{AE} = 0$

$F_{AE} = 4.62$ kN 1

Consider joint B



$\Sigma V = 0$

$8 + F_{AB} \sin 60^\circ + F_{BE} \sin 60^\circ = 0$

$8 + (-9.24)\,0.866 + F_{BE} \times 0.866 = 0$ 1

$F_{BE} = 0 = F_{CE}$

$\Sigma H = 0$

$F_{BC} + F_{BE} \cos 60^\circ - F_{AB} \cos 60^\circ = 0$

$F_{BC} + 0 \times 0.5 - (-9.24) \times 0.5 = 0$ 1

$F_{BC} = -4.62$ kN.

Results

$F_{AB} = F_{CD} = 9.24$ Compression

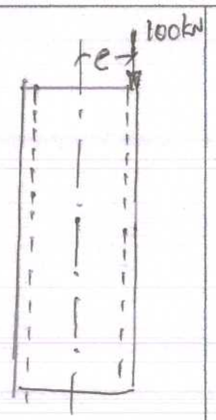
$F_{BC} = \cancel{F_{DE}} = 4.62$ Compression

$F_{AE} = F_{DE} = 4.62$ Tension 2 7

$F_{BE} = F_{CE} = 0$

V (a)

$D = 300\ mm$, $d = 300 - (2 \times 25)$
$= 250\ mm$



$A = \frac{\pi}{4}(D^2 - d^2)$

$= \frac{\pi}{4}(300^2 - 250^2)$

$= 21.6 \times 10^3\ mm^2$ 2

$Z = \frac{\pi}{32}\left[\frac{D^4 - d^4}{D}\right]$

$= \frac{\pi}{32}\left[\frac{300^4 - 250^4}{300}\right]$ 2

$= 1372 \times 10^3\ mm^3$

$e = 150\ mm$ ($\because$ load at outer edge)

$M = P.e = 100 \times 10^3 \times 150 = 15 \times 10^6\ N\text{-}mm$

$\therefore \sigma_{max} = \frac{P}{A} + \frac{M}{Z} = \frac{100 \times 10^3}{21.6 \times 10^3} + \frac{15 \times 10^6}{1372 \times 10^3}$ 2

$= \underline{\underline{15.56\ MPa}}$

$\sigma_{min} = \frac{P}{A} - \frac{M}{Z} = \frac{100 \times 10^3}{21.6 \times 10^3} - \frac{15 \times 10^6}{1372 \times 10^3}$

$= \underline{\underline{-6.3\ MPa}}$ (Tension) 2 8

V b) Sol.

$H = 15\ m$, $h = 10\ m$, $b = 6\ m$, $w = 10\ kN/m^3$

i) Total pressure per metre length of the dam.

$P = \frac{wh^2}{2} = \frac{10 \times 10^2}{2} = 500\ kN$ 1

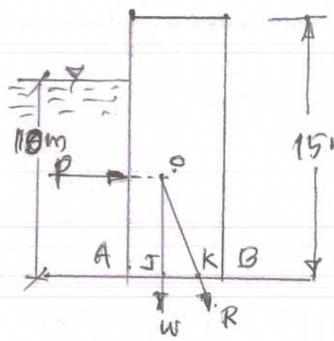
ii) point where the resultant cuts

weight of the concrete per metre length

$W = \rho \times b \times H$, $\rho = 25\ kN/m^3$

$W = 25 \times 6 \times 15 = 2250\ kN$ 1

Horizontal distance between C.G of the dam section and the point where the resultant cuts the base (JK)



$$x = \frac{P}{W} \times \frac{h}{3}$$

$$= \frac{500}{2250} \times \frac{10}{3} = 0.741\ m.$$

$e = x = \underline{0.741\ m}$ 1

iii) Maximum and minimum intensities of stresses

$$\sigma_{max} = \frac{W}{b}\left[1 + \frac{6e}{b}\right] = \frac{2250}{6}\left[1 + \frac{6 \times 0.741}{6}\right]$$ 1½

$\sigma_{max} = \underline{652.875\ kN/m^2}$ (compression)

$$\sigma_{min} = \frac{W}{b}\left[1 - \frac{6e}{b}\right] = \frac{2250}{6}\left[1 - \frac{6 \times 0.741}{6}\right]$$ 1½

1 Fig 7

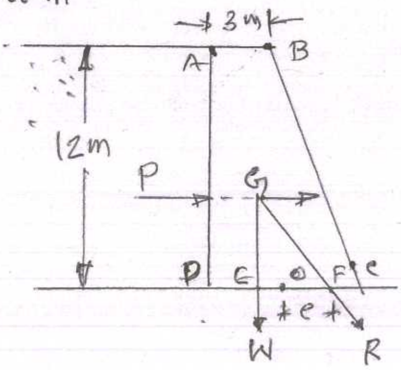
$\sigma_{min} = \underline{97.13\ kN/m^2}$

VI a) Sol.

$h = 12\ m$, $a = 3\ m$, $b = 6\ m$, $\phi = 30°$, $w = 20\ kN/m^3$

$\rho = 25\ kN/m^3$

Horizontal thrust on the retaining wall per metre length



1 Fig

$$P = \frac{wh^2}{2} \times \frac{1 - \sin\phi}{1 + \sin\phi}$$

$$= \frac{20 \times 12^2}{2} \times \frac{1 - \sin 30°}{1 + \sin 30°}$$

$P = 480\ kN$ 1

Weight of masonry wall per metre length

$W = \left[\frac{6+3}{2}\right] 12 \times 1 \times 25 = 1350\ kN$ 1

$DE = \dfrac{a^2+ab+b^2}{3(a+b)} = \dfrac{3^2+3\times6+6^2}{3(3+6)} = 2.33\,m$ 1

$FF = \dfrac{P}{W}\times\dfrac{h}{3} = \dfrac{480}{1350}\times\dfrac{12}{3} = 1.42\,m$ 1

$DF = 2.33+1.42 = 3.75\,m$

eccentricity, $e = DF - \dfrac{b}{2} = 3.75 - \dfrac{6}{2} = 0.75\,m$

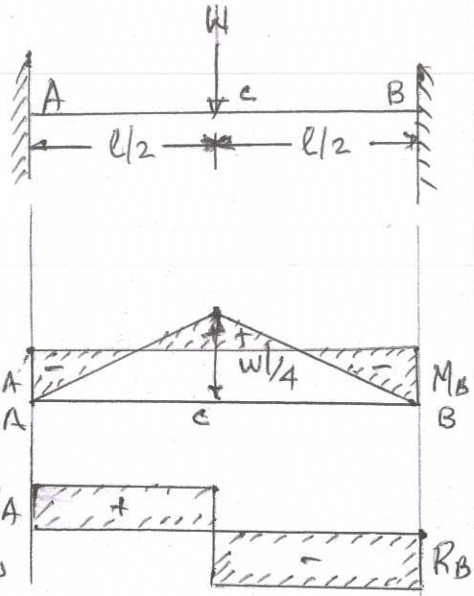
$\sigma_{max} = \dfrac{W}{b}\left[1+\dfrac{6e}{b}\right] = \dfrac{1350}{6}\left[1+\dfrac{6\times0.75}{6}\right] = 393.75$ 1½

$\sigma_{max} = \underline{\underline{393.75\ kN/m^2}}$

$\sigma_{min} = \dfrac{W}{b}\left[1-\dfrac{6e}{b}\right] = \dfrac{1350}{6}\left[1-\dfrac{6\times0.75}{6}\right] = 56.25$ 1½

$\sigma_{min} = \underline{\underline{56.25\ kN/m^2}}$ 8

VI b) Fixed beam with central load



i) B.M. diagram

M_A = Fixing moment at A

M_B = Fixing moment at B

Let M = BM diagram due to point load

M' = BM diagram due to Fixing moments 1½

Equating areas of two diagrams 1½

Area of M diagram = Area of M' diagram

$M_A \cdot l = -\dfrac{1}{2}\times l \cdot \dfrac{Wl}{4}$

$M_A = \underline{\underline{\dfrac{-Wl}{8}}}$ ||ly $M_B = \underline{\underline{\dfrac{-Wl}{8}}}$ 2

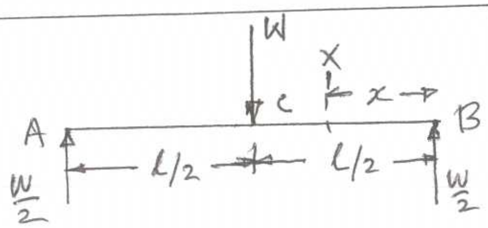
ii) Shear force diagram

$R_B \times l + M_A = M_B \times W \times \dfrac{l}{2}$

$\therefore R_B = \underline{\underline{\dfrac{W}{2}}}$ ||ly $R_A = \underline{\underline{\dfrac{W}{2}}}$ 2 7

VII

a) Simply Supported beam AB of span l and central point load W.



$R_A = R_B = \frac{W}{2}$

Consider a Section X at a distance x from B

$M_X = R_B \cdot x = \frac{W}{2}x = \frac{Wx}{2}$

$\therefore EI \frac{d^2y}{dx^2} = \frac{Wx}{2}$ --- i 1

$EI \frac{dy}{dx} = \frac{Wx^2}{4} + C_1$ --- ii 1

Where C_1 is the constant of Integration

When $x = \frac{l}{2}$, $\frac{dy}{dx} = 0$, substituting

$0 = \frac{Wl^2}{16} + C_1$ or $C_1 = -\frac{Wl^2}{16}$ 1

Substituting in eqn- ii

$EI \frac{dy}{dx} = \frac{Wx^2}{4} - \frac{Wl^2}{16}$ --- iii) 1

for max. slope, Put $x = 0$ and $\frac{dy}{dx} = i_B$

$EI\, i_B = \frac{-Wl^2}{16}$ or $i_B = \frac{-Wl^2}{16EI}$ radians

$i_B = i_A = \frac{-Wl^2}{16EI}$ radians 1

Integrating Eqn. iii again

$EIy = \frac{Wx^3}{12} - \frac{Wl^2}{16}x + C_2$

substitute when $x = 0$, $y = 0$ 1

$0 = 0 - 0 + C_2$, $C_2 = 0$

$\therefore EIy = \frac{Wx^3}{12} - \frac{Wl^2}{16}x$ — General equation

For max deflection put $x = \frac{l}{2}$ 1

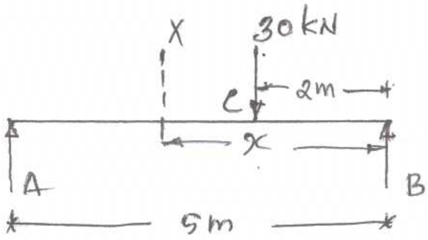
$EIy_c = \frac{W}{12}\left(\frac{l}{2}\right)^3 - \frac{Wl^2}{16}\left(\frac{l}{2}\right)$

$y_c = -Wl^3/48EI$ 1 8

VII b)

Sol.

$EI = 40 \times 10^3 \ kN\text{-}m^2$



$R_A = \frac{30 \times 2}{5} = 12 \ kN.$

$R_B = 30 - 12 = 18 \ kN$ 1

Consider a section X at a distance x from support B

$BM \text{ at } X = R_B x - 30(x-2)$

$EI \frac{d^2y}{dx^2} = R_B x - 30(x-2)$ 1

$EI \frac{dy}{dx} = \frac{R_B x^2}{2} - \frac{30(x-2)^2}{2} + C_1$

$= \frac{18x^2}{2} - \frac{30(x-2)^2}{2} + C_1 \quad \text{- - - (i)}$ 1

once again Integrating

$EI y = \frac{18x^3}{3 \times 2} - \frac{30(x-2)^3}{3 \times 2} + C_1 x + C_2 \quad \text{- - - (ii)}$ 1

Put when $x = 0$, $y = 0$, $C_2 = 0$ and $x = 5$, $y = 0$

substitute in eqn (ii)

$0 = \frac{18 \times 5^3}{6} - \frac{30(5-2)^3}{6} + 5C_1$

$C_1 = -48 \ kN/m^2$ 1

Substitute the value of C_1 in eqn-(ii) and for deflection under load, Put $x = 2$

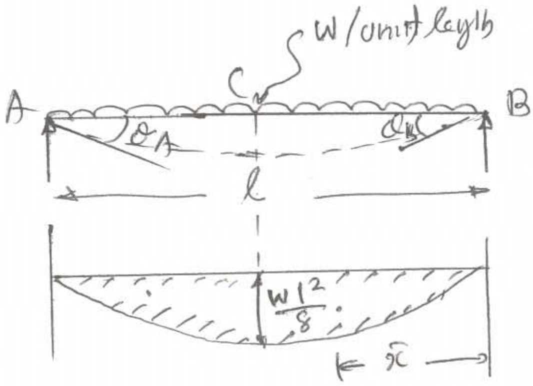
$EI y = \frac{18 \times 2^3}{6} - \frac{30(2-2)^3}{6} + -48 \times 2$

$y = \frac{1}{EI}\left[24 - 96\right] = \frac{1}{40 \times 10^3} \times -72$

$y = \underline{\underline{3 \ mm}}$ 2 7

VIII a) Sol.

Area of BM diagram between C and B



$= \frac{Wl^2}{8} \times \frac{l}{2} \times \frac{2}{3}$ — 2 Fig

$= \frac{Wl^3}{24}$

Distance between C.G of BM diagram between C and B to B

$\bar{x} = \frac{5}{8} \times \frac{l}{2} = \frac{5l}{16}$ — 1

According to mohrs Theoram I

$\theta_B = \frac{A}{EI} = \frac{Wl^3}{24EI} = \theta_A$ — 2

$\theta_A = \theta_B = \frac{Wl^3}{24EI}$ radians

According to mohrs Theoram - II

$y_c = \frac{A\bar{x}}{EI} = \frac{Wl^3}{24EI} \times \frac{5l}{16} = \frac{5Wl^4}{384EI}$ — 1+2 — 8

VIII b) Sol.

$l = 3$ m — 1

slope at ends $= 1^\circ = 0.01745$ radian

slope at the end for SS beam with Central load $\Big\}$ $\theta = \frac{Wl^2}{16EI}$ — 1

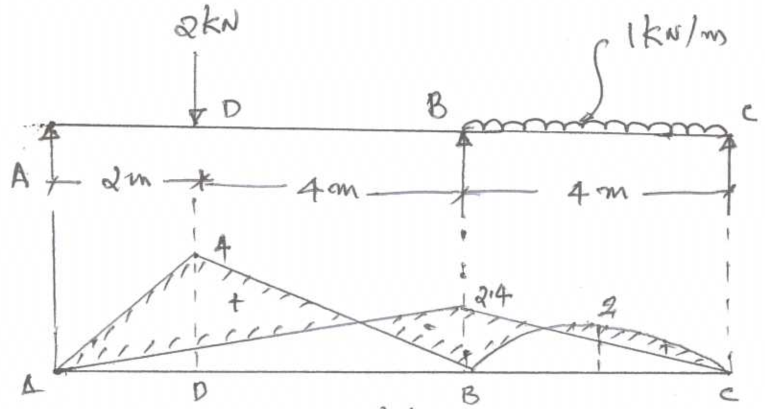
$\therefore \frac{Wl^2}{16EI} = 0.01745$ — 1

Deflection at the Centre, $y_c = \frac{Wl^3}{48EI} = \frac{Wl^2}{16EI} \times \frac{l}{3}$ — 2

$y_c = 0.01745 \times \frac{3}{3}$

$y_c = 1.745$ cm — 2 — 7

IX a)



2 Fig

Consider beam AB as simply supported

$$M_D = \frac{Wab}{L_1} = \frac{3\times2\times4}{6} = 4\text{ kN-m}$$ 1

IIly span BC

$$M_C = \frac{Wl_2^2}{8} = \frac{1\times4^2}{8} = 2\text{ kN-m}$$ 1

$$a_1\bar{x}_1 = \left[\left(\tfrac{1}{2}\times2\times4\times\tfrac{2\times2}{3}\right)+\left(\tfrac{1}{2}\times4\times4\right)\left(2+\tfrac{4}{3}\right)\right]$$ 1

$$= 32$$

$$a_2\bar{x}_2 = \left[\tfrac{2}{3}\times2\times4\times2\right] = \frac{32}{3}$$ 1

using three moments equation

$$M_A l_1 + 2M_B(l_1+l_2) + M_C l_2 = -\left[\frac{6a_1x_1}{l_1} + \frac{6a_2\bar{x}_2}{l_2}\right]$$ 1

$M_A = 0$, ~~$M_B = 0$~~ $M_C = 0$

$$0 + 2M_B(6+4) + 0 = \left(\frac{6\times32}{6} + \frac{6\times\frac{32}{3}}{4}\right)$$

$$20M_B = (-(32+16))$$

$$\therefore M_B = -\frac{48}{20} = \underline{\underline{-2.4\text{ kN-m}}}$$ 2 9

IX b) i) Distributed moments :- The unbalanced moments cause the joints to rotate. The resisting moments developed in the members are called distributed moments. 2

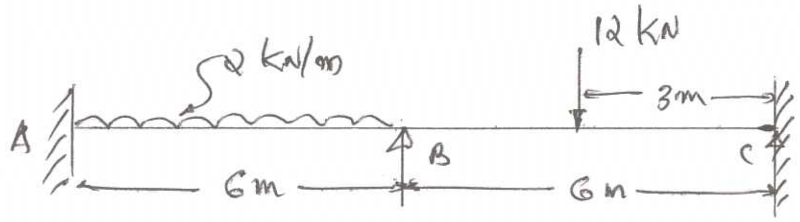
ii) Unbalanced moments : The algebraic sum of the moments meeting at a joint is called unbalanced moment 2

iii) Carry over moment : The moment induced at the far end due to the distributed moment at the nearer end, the far end being fixed is known as carry over moment. 2 6

X a) Clapeyron's theoram states, ib a beam has 'n' supports, the end ones being fixed, then the same number of equations required to determine the support moments may be obtained from the consecutive pairs of spans ie AB - BC, BC - CD, CD - DE and so on. 3

$$M_A\left(\frac{l_1}{I_1}\right) + 2M_B\left[\frac{l_1}{I_1} + \frac{l_2}{I_2}\right] + M_C\left(\frac{l_2}{I_2}\right) = -\left[\frac{6a_1\bar{x}_1}{I_1 l_1} + \frac{6a_2\bar{x}_2}{I_2 l_2}\right]$$ 3 6

X b)



Consider Continuous beam ABC, split in to two fixed beams AB and BC

For span AB, Fixing momet at A = $-\frac{wl^2}{12} = \frac{2\times 6^2}{12} = -6.00$ 1

" Fixing moment at B = $+\frac{wl^2}{12} = +6.00$ kN-m. 1

For span BC, Fixing moment at B = $-\frac{wl}{8}$

$= \frac{-12\times 6}{8} = -9$ kN-m. 1

Fixing moment at C = $+\frac{wl}{8} = 9$ kN-m. 1

stiffness factor at joint B

$K_{BA} = \frac{4EI}{l} = \frac{4EI}{6} = \frac{2EI}{3}$

$K_{BC} = \frac{4EI}{l} = \frac{4EI}{6} = \frac{2EI}{3}$

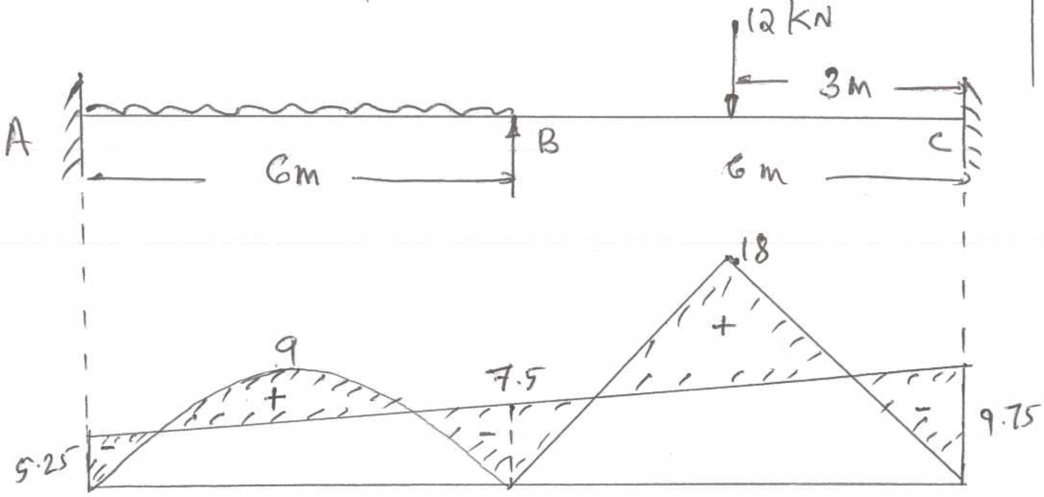
At joint B, the Distribution factors are

$DF_{BA} = \frac{K_{BA}}{\Sigma K_B} = \frac{2EI}{3} \times \frac{3}{4EI} = \frac{1}{2}$ 1

$DF_{BC} = \frac{K_{BC}}{\Sigma K_B} = \frac{2EI}{3} \times \frac{3}{4EI} = \frac{1}{2}$ 1

Joint	A		B		C
Distribution factor			$\frac{1}{2}$	$\frac{1}{2}$	
Fixed end moments	-6.0	+6.0		-9.0	+9.0
		+1.5		+1.5	
Carry over	+0.75				+0.75
Distribute	0	0		0	0
Final moments	-5.25	+7.5		-7.5	+9.75

2



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