

20 D

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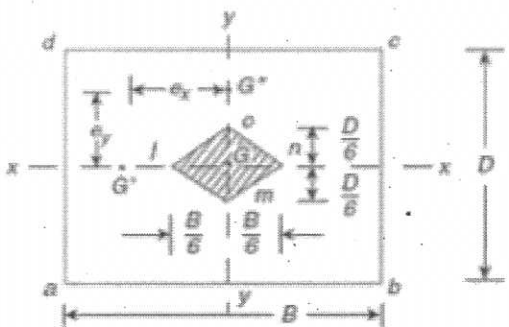
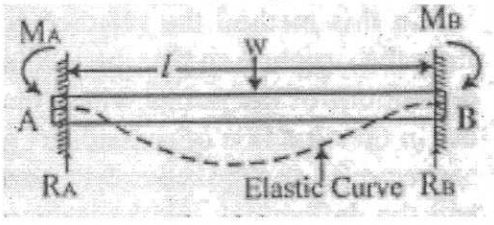
Revision : 15

SCORING INDICATORS

CODE : 4014 - THEORY OF STRUCTURES - II

VERSION: Revision 2015

Qn.No		Split score	Total score
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Qn.No	PART-A	Split score	Total score
I. 1	Slenderness ratio (λ) is the ratio of the effective length of a column (L_e) and the least radius of gyration (r) about the axis under consideration.	2	
2	Limit of eccentricity of rectangular column sections 	2	
3		2	
4	$\theta_B = \frac{-Wl^2}{16EI} \text{ Radians} \quad \theta_A = \frac{Wl^2}{16EI} \text{ Radians}$ $Y_C = \frac{-Wl^3}{48EI}$	2	
5	The distribution factor for a member at a joint is the ratio of the stiffness factor of the member to the total stiffness factor of all the members meeting at the joint.	2	10
II) 1	<p style="text-align: center;"><u>PART - B</u></p> <p><u>SOLUTION</u> $l = 5 \text{ m} = 5 \times 10^3 \text{ mm}$ $d = 40 \text{ mm}$ $E = 200 \text{ GPa} = 200 \times 10^3 \text{ N/mm}^2$ moment of inertia, $I = \frac{\pi}{64} d^4$</p>	1	

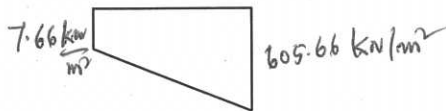
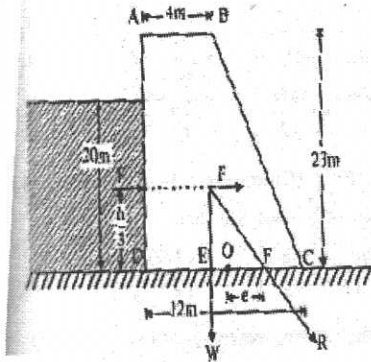
$$= \frac{\pi}{64} (40)^2 = 40000\pi \text{ mm}^4 = 125600 \text{ mm}^4$$

Since the column is fixed at one end and other end free

$$\therefore L_e = 2l = 2 \times 5 \times 10^3 = 10 \times 10^3 \text{ mm}$$

$$\begin{aligned} \therefore \text{Eulers Crippling load, } P_E &= \frac{\pi^2 EI}{L_e^2} \\ &= \frac{\pi^2 (200 \times 10^3) \times 40000\pi}{(10 \times 10^3)^2} \\ &= \underline{\underline{2.48 \text{ kN}}} \end{aligned}$$

Solution-Given



$$H = 23\text{m}, h = 20\text{m}, b = 12\text{m},$$

$$a = 4\text{m}$$

$$w = 10\text{kN/m}^3$$

The total pressure of water on the dam per unit length

$$\frac{wh^2}{2} = \frac{10 \times 20^2}{2} = 2000\text{kN}$$

Weight of the dam per unit length

$$W = \frac{(4+12)}{2} \times 23 \times 1 \times 20 = 3680\text{kN}$$

$$\begin{aligned} DE &= \frac{4 \times 23 \times 2 + \frac{8 \times 23}{2} \left(4 + \frac{8}{3}\right)}{4 \times 23 + \frac{8 \times 23}{2}} \\ &= 4.33\text{m}. \end{aligned}$$

Stress
diagram

The resultant R cuts the base DC at F

$$EF = \frac{F}{W} \times \frac{h}{3} = \frac{2000}{3680} \times \frac{20}{3} = 3.62 \text{ m}$$

$$DF = DE + EF = 4.33 + 3.62 = 7.95 \text{ m}$$

The above weight W acts through the centre of gravity of the section and cuts the base DC at E.

$$\begin{aligned} \text{eccentricity of the resultant, } e &= DF - \frac{b}{2} \\ &= 7.95 - \frac{12}{2} = 1.95 \text{ m} \end{aligned}$$

Maximum stress occurs at the edge C of the base

$$f_{\text{max}} = \frac{W}{b} \left[1 + \frac{6e}{b} \right]$$

$$= \frac{3680}{12} \left[1 + \frac{6 \times 1.95}{12} \right] = 605.66 \text{ kN/m}^2 \text{ (Compressive) -}$$

Minimum stress occurs at the edge D of the base

$$f_{\text{min}} = \frac{W}{b} \left[1 - \frac{6e}{b} \right]$$

$$= \frac{3680}{12} \left[1 - \frac{6 \times 1.95}{12} \right]$$

$$= 7.66 \text{ kN/m}^2 \text{ (Compressive) - -}$$

3

1. Angle of repose

The minimum angle of the plane at which the body kept on it starts to slide due to its own weight is called angle of repose. The body will begin to move down the plane, if the angle of inclination of the plane is equal to the angle of friction.

2

2. Weep Holes

Provided in earth retaining structures like retaining walls, underpasses, wing walls and other below ground drainage structures.

Weep Hole is provided in these structures to relieve hydrostatic pressure or water pressure on the walls.

Reducing the water pressure on the walls will reduce the structural design demand of the water or earth resisting wall by reducing its thickness as well as reinforcement requirements.

2

3. Active earth pressure

The minimum value of lateral earth pressure exerted by soil on a structure and the wall moves away from the backfill, occurring when the soil is allowed to yield sufficiently to cause its internal shearing resistance along a potential failure surface to be completely mobilized.

2

Solution-given

$$l = 6\text{m} = 6000\text{mm}$$

$$I = 300 \times 10^6 \text{mm}^4$$

$$Y_c = 4\text{mm}$$

$$Y_c = \frac{5wl^4}{384EI}$$

$$\frac{5 \times w \times 6000^4}{384 \times 200 \times 10^3 \times 300 \times 10^6}$$

$$4 = 0.281w$$

$$w = 14.2\text{kN/m}$$

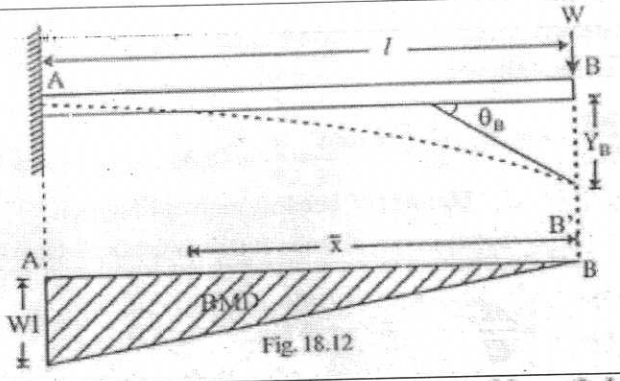
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6

5



Consider a cantilever AB of length l carrying a point load W

Area of bending moment diagram

$$A = \frac{wl \times l}{2} = \frac{wl^2}{2}$$

Distance between CG of bending moment diagram and B

$$\bar{x} = \frac{2l}{3}$$

According to Mohr's theorem I, change of slope between A and B is

$$\theta_B = \frac{A}{EI} = \frac{Wl^2}{2EI} \text{ radians}$$

According to Mohr's theorem II, intercept of the tangents on A and B is

$$BB' = Y_B = \frac{A\bar{x}}{EI} = \frac{Wl^2}{2EI} \times \frac{2l}{3} = \frac{Wl^3}{3EI}$$

6

The theorem of three moments gives the relationship between the moments at supports in a continuous beam. The theorem states that if AB and BC are any two consecutive spans of a continuous beam subjected to an external loading, the support moment M_a, M_b and M_c are given by the relation.

$$M_a l_1 + 2M_b (l_1 + l_2) + M_c l_2 = \frac{6a_1 \bar{x}_1}{l_1} + \frac{6a_2 \bar{x}_2}{l_2}$$

7

1. Stiffness Factor- i) It is the moment that must be applied at one end of a constant section member (which is unyielding supports at both ends) to produce a unit rotation of that end when the other end is fixed, i.e. $k = 4EI/l$

ii) It is the moment required to rotate the near end of a prismatic member through a unit angle without translation, the far end being hinged is $k = 3EI/l$.

2. Carry Over Factor

It is the ratio of induced moment to the applied moment

The ratio of moment produced at a joint to the moment applied at the other joint without displacing is called carry over factor.

The carry over factor is always (1/2) for members of constant moment of inertia. If the end is hinged/pin connected, the carry over factor is zero.

3. unbalanced moment :- the sum of the fixed end moments meeting at a joint which is initially clamped is not zero. The algebraic sum of the moments meeting at a joint is called unbalanced moment.

III (a)

PART-C

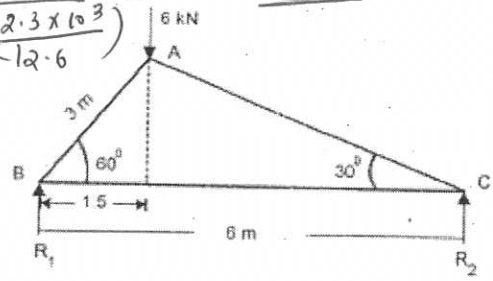
Solution $D = 38 \text{ mm}$, Thickness = 2.5 mm and $d = 38 - 5 = 33 \text{ mm}$
 $l = 2.3 \times 10^3 \text{ mm}$, $\sigma_c = 335 \text{ N/mm}^2$, Rankines constant $a = \frac{1}{7500}$
 $I = \frac{\pi}{64} (D^4 - d^4) = \frac{\pi}{64} (38^4 - 33^4) = 14.05 \times 10^3 \pi \text{ mm}^4$
 Both ends hinged, $l_e = l \times 2.3 \times 10^3 \text{ mm}$
 $P_E = \frac{\pi^2 E I}{l_e^2} = \frac{\pi^2 (205 \times 10^3) \times (14.05 \times 10^3 \pi)}{(2.3 \times 10^3)^2} = 16.88 \text{ kN}$

Rankines crippling Load $P_R = \frac{\sigma_c A}{1 + a(l_e/k)^2}$

$k = \sqrt{\frac{I}{A}}$
 $A = \frac{\pi}{4} (D^2 - d^2) = \frac{\pi}{4} (38^2 - 33^2) = 88.75 \pi \text{ mm}^2$
 $k = \sqrt{\frac{14.05 \times 10^3 \pi}{88.75 \pi}} = 12.6 \text{ mm}$

$P_R = \frac{335 \times 88.75 \pi}{1 + \frac{1}{7500} \left(\frac{2.3 \times 10^3}{12.6} \right)^2} = 17.16 \text{ kN}$

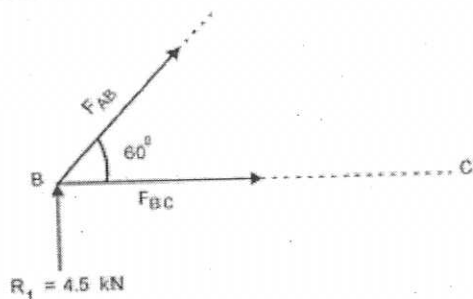
III (b)



Taking moments about C

$R_1 \times 6 = 6 \times 4.5 \Rightarrow R_1 = 4.5 \text{ kN}$
 and $R_2 = 6 - 4.5 = 1.5 \text{ kN}$

Joint B



$$R_1 = 4.5 \text{ kN}$$

Resolving vertically, $\Sigma V = 0$

$$4.5 + F_{BA} \sin 60^\circ = 0$$

$$F_{BA} = -\frac{4.5}{\sin 60^\circ} = -5.196 \text{ kN.}$$

$$\therefore F_{BA} = F_{AB} = 5.196 \text{ kN (compressive)}$$

Resolving horizontally, $\Sigma H = 0$

$$F_{BA} \cos 60^\circ + F_{BC} = 0$$

$$-5.196 \cos 60^\circ + F_{BC} = 0$$

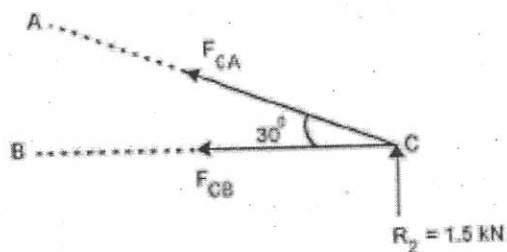
$$F_{BC} = +2.598 \text{ kN}$$

$$\therefore F_{BC} = F_{CB} = 2.598 \text{ kN (tensile).}$$

1/2

1/2

Joint C



Here, $\Sigma V = 0$

$$F_{CA} \sin 30^\circ + 1.5 = 0$$

$$F_{CA} = -3.0 \text{ kN}$$

$$F_{CA} = F_{AC} = 3.0 \text{ kN (compressive)}$$

1/2

$$F_{CA} \cos 30^\circ + F_{CB} = 0$$

$$-3 \cos 30^\circ + F_{CB} = 0$$

$$F_{CB} = 2.598 \text{ kN} = F_{BC}$$

Results :

Joint	Member	Force (kN)
A	AB	-5.196
	AC	-3.0
B	BA	-5.196
	BC	+2.598
C	CA	-3.0
	CB	2.598

1/2

7

IV

(a)

Solution - Given

$$l = 12 \text{ m} = 1200 \text{ cm}$$

$$A = 1 \times 1 = 1 \text{ m}^2 = 10^4 \text{ cm}^2$$

$$E = 2 \times 10^4 \text{ KN/cm}^2$$

- a) Both ends of the column are pinned
Buckling load,

$$P_E = \frac{\pi^2 EI}{L^2} \quad \text{where } L = l$$

2

$$P_E = \frac{\pi^2 \times 2 \times 10^4 \times \frac{100 \times 100^3}{12}}{1200^2}$$

$$= 1142310.8 \text{ kN}$$

2

- b) One end fixed and other end free

Buckling load, $P_E = \frac{\pi^2 EI}{L^2}$ where $eL = 2l$

2

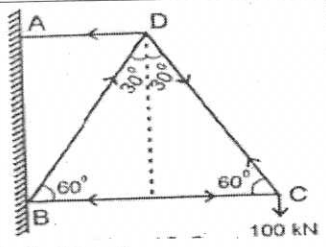
$$P_E = \frac{\pi^2 \times 2 \times 10^4 \times \frac{100 \times 100^3}{12}}{(2 \times 1200)^2}$$

$$= 285577.7 \text{ kN}$$

2

8

(V. b)



Solution-

joint C, $\sum H = 0, \sum V = 0$

$$P_{CD} \sin 60 = 100$$

$$P_{CD} = \frac{100}{\sin 60} = 115.47 \text{ kN (tension)}$$

$$P_{CB} = P_{CD} \cos 60 = 115.47 \cos 60 = 57.73 \text{ kN (Compression)}$$

Consider the equilibrium of the joint D, $\sum H = 0, \sum V = 0$

$$P_{DC} \cos 30 = P_{DB} \cos 30$$

$$P_{DC} = P_{DB} = 115.47 \text{ kN (Compression)}$$

$$P_{DA} = P_{DC} \sin 30 + P_{DB}$$

$$\sin 30 = 115.47 \sin 30 + 115.47 \sin 30 = 115.47 \text{ kN (Tension)}$$

1/2

1/2

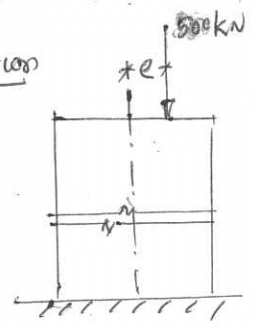
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2/2

7

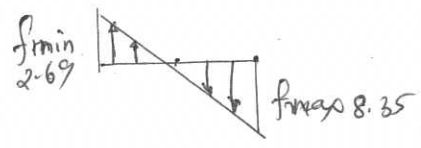
V (a)

Solution



- D = 25 cm
- d = 20 cm
- e = 10 cm
- P = 500 kN

fig 1



$$\text{Direct strain} = \frac{P}{A}$$

$$= \frac{500 \times 4}{\pi(25^2 - 20^2)} = 2.83 \text{ kN/cm}^2$$

$$\text{Bending strain} = \frac{m \times y}{I} = \frac{P \cdot e \cdot y}{I} = \frac{500 \times 10 \times 12.5 \times 64}{\pi(25^4 - 20^4)} = 5.52 \text{ kN/cm}^2$$

$$f_{\max} = f_d + f_b = 2.83 + 5.52 = 8.35 \text{ kN/cm}^2$$

$$f_{\min} = f_d - f_b = 2.83 - 5.52 = -2.69 \text{ kN/cm}^2$$

f_{\max} = Compression, f_{\min} = tensile

2

2

1

1

1

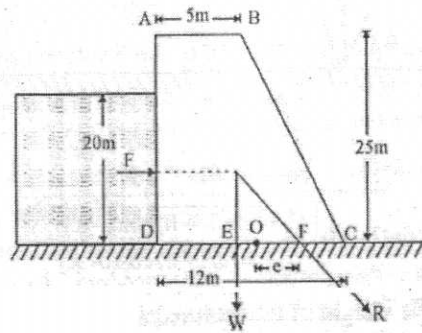
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V

(b)

Solution-Given

$$H=25\text{m}, b=12\text{m}, a=5\text{m}, P=25\text{kN/m}^3, w=10\text{kN/m}^3$$



Weight of the dam per unit length

$$W = \text{specific weight} \times \text{volume}$$

$$= 25 \times \left(\frac{12+5}{2} \right) 25 \times 1$$

$$= 5312.5 \text{ kN}$$

Total pressure of water per unit length of the dam

$$F = \text{Average intensity of pressure} \times \text{Area}$$

$$= \left(\frac{wh+0}{2} \right) h \times 1 = \frac{wh^2}{2} = \frac{10 \times 20^2}{2} = 2000 \text{ kN}$$

i) Resultant pressure on the base per metre length of the dam

$$R = \sqrt{F^2 + W^2} = \sqrt{5312.5^2 + 2000^2} = 5676 \text{ kN} \text{ -----}$$

ii) Take moment about F, $F \times \frac{h}{3} = W \times EF$

$$EF = \frac{F}{W} \times \frac{h}{3} = \frac{2000}{5312.5} \times \frac{20}{3} = 2.5 \text{ m}$$

$$DE = \frac{5 \times 25 \times 2.5 + \frac{1}{2} \times 7 \times 25 \left(5 + \frac{1}{3} \right)}{5 \times 25 + \frac{1}{2} \times 7 \times 25} = 4.49 \text{ m}$$

$$DF = DE + EF = 4.49 + 2.5 = 6.99 \text{ m}$$

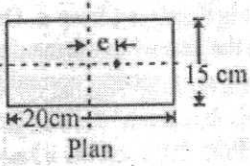
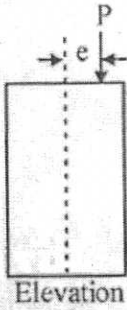
The resultant R cuts the base at a distance 6.99 m from D.

$$\text{iii) eccentricity of the resultant, } e = DF - \frac{b}{2}$$

$$= 6.99 - \frac{12}{2} = 0.99 \text{ m}$$

VI

(a)



Maximum stress, $f_{max} = \frac{P}{A} + \frac{Mx y}{I}$

$$= \frac{1000}{20 \times 15} + \frac{1000 \times 5 \times 10 \times 12}{15 \times 20^3}$$

$$= 8.33 \text{ kN/cm}^2$$

Minimum stress, $f_{min} = \frac{P}{A} - \frac{Mx y}{I}$

$$= \frac{1000}{20 \times 15} - \frac{1000 \times 5 \times 10 \times 12}{15 \times 20^3}$$

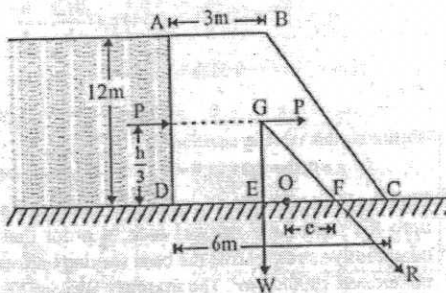
$$= -1.67 \text{ kN/cm}^2$$

1
2
1
2
1

8

VI

(b)



Solution - Given

$h = 12\text{m}, a = 3\text{m}, b = 6\text{m}, \phi = 30^\circ, w = 20\text{kN/m}^3, \rho = 25\text{kN/m}^3$

Horizontal thrust on the retaining wall per metre length.

$$P = \frac{wh^2}{2} \times \frac{1 - \sin\phi}{1 + \sin\phi} = \frac{20 \times 12^2}{2} \times \frac{1 - \sin 30^\circ}{1 + \sin 30^\circ} = 480 \text{ kN}$$

1

Weight of masonry wall per metre length

$$W = \left(\frac{6+3}{2} \right) 12 \times 1 \times 25 = 1350 \text{ kN}$$

Horizontal distance of the CG of the retaining wall from the vertical face AD

$$DE = \frac{a^2 + ab + b^2}{3(a+b)} = \frac{3^2 + 3 \times 6 + 6^2}{3(3+6)} = 2.33 \text{ m}$$

$$EF = \frac{P}{w} \times \frac{h}{3} = \frac{480}{1350} \times \frac{12}{3} = 1.42 \text{ m}$$

$$DF = DE + EF = 2.33 + 1.42 = 3.75 \text{ m}$$

$$\begin{aligned} \text{eccentricity of the resultant, } e &= DF - \frac{b}{2} \\ &= 3.75 - \frac{6}{2} = 0.75 \text{ m} \end{aligned}$$

Maximum stress occurs at the edge C of the base

$$\begin{aligned} f_{\text{max}} &= \frac{W}{b} \left[1 + \frac{6e}{b} \right] = \frac{1350}{6} \left[1 + \frac{6 \times 0.75}{6} \right] \\ &= 393.75 \text{ kN/m}^2 \text{ (Compressive)} \end{aligned}$$

Minimum stress occurs at the edge D of the base

$$\begin{aligned} f_{\text{min}} &= \frac{W}{b} \left[1 - \frac{6e}{b} \right] = \frac{1350}{6} \left[1 - \frac{6 \times 0.75}{6} \right] \\ &= 56.25 \text{ kN/m}^2 \text{ (Compressive)} \end{aligned}$$

VII
(a)

Solution

$$\text{span } l = 4 \text{ m} = 4 \times 10^3 \text{ mm}$$

$$\text{UDL, } w = 2 \text{ kN/m}$$

$$E = 200 \text{ GPa} = 200 \times 10^3 \text{ N/mm}^2$$

$$I = 400 \times 10^3 \text{ mm}^4$$

$$\begin{aligned} \text{maximum slope} &= \theta_A = \frac{wl^3}{24EI} \\ &= \frac{2 \times (4 \times 10^3)^3}{24 \times 200 \times 10^3 \times 400 \times 10^3} \end{aligned}$$

Slope $\theta_A = 0.067 \text{ rad}$.

Maximum deflection

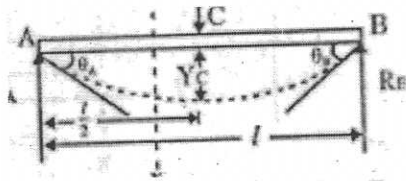
$$y_c = \frac{5Wl^4}{384EI} = \frac{5 \times 2 \times (4 \times 10^3)^4}{384 \times 200 \times 10^3 \times 400 \times 10^3}$$

$$= 83.33 \text{ mm}$$

VII
(b)

Simply supported beam with a central point load

Consider a simply supported beam AB of length 'l' and carrying a point load W at the centre of the beam:



Consider a section XX at a distance x from B

$$\text{BM at section xx, } M_x = \frac{Wx}{2} - W(x - \frac{l}{2})$$

According to the differential equation of flexure

$$EI \frac{d^2y}{dx^2} = M_x$$

$$EI \frac{d^2y}{dx^2} = \frac{Wx}{2} - W(x - \frac{l}{2})$$

$$EI \frac{dy}{dx} = \frac{Wx^2}{4} + C_1 - \frac{W}{2} (x - \frac{l}{2})^2 \text{-----(1)}$$

$$EIY = \frac{Wx^3}{12} + C_1x + C_2 - \frac{W}{6} (x - \frac{l}{2})^3 \text{-----(2)}$$

where C_1 and C_2 are the constant of integration

When $x=0$, $y=0$ and consider terms up to dotted line in equation (2), we get $C_2=0$, When $x=l$, $y=0$ and consider all the terms in equation(2)

$$0 = \frac{Wl^3}{12} + C_1l - \frac{W}{6} (l - \frac{l}{2})^3$$

$$C_1l = \frac{-Wl^3}{12} + \frac{W}{6} (\frac{l}{2})^3$$

$$= \frac{-Wl^3}{12} + \frac{Wl^3}{48} = \frac{-Wl^3}{16}$$

$$C_1 = \frac{-Wl^2}{16}$$

Substituting this value of C_1 in equation (1)

$$EI \frac{dy}{dx} = \frac{Wx^2}{4} - \frac{Wl^2}{16} - \frac{W}{2} (x - \frac{l}{2})^2 \dots\dots\dots(3)$$

This is the required equation for slope at any section.

When $x = 0$, $\theta_B = \frac{-Wl^2}{16EI}$ radian

When $x = l$, $\theta_A = \frac{Wl^2}{16EI}$ radian

Substituting the value of C_1 and C_2 in equation (2)

$$EIY = \frac{Wx^3}{12} + \frac{Wl^2x}{16} - \frac{W}{6} (x - \frac{l}{2})^3 \dots\dots\dots(4)$$

This is the required equation for deflection at any section.

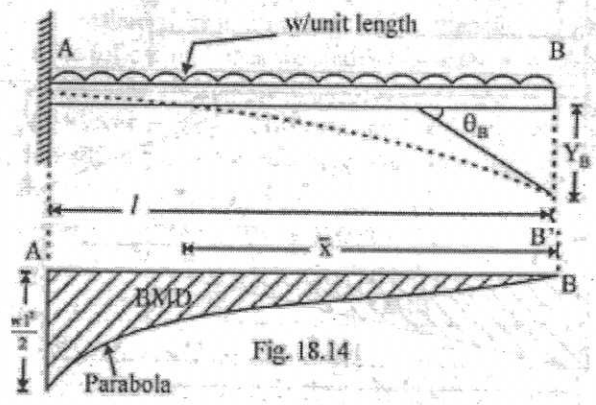
Substituting $x = \frac{l}{2}$ in the equation (4) and consider terms upto dotted line.

$$EIY_c = \frac{W}{12} (\frac{l}{2})^3 - \frac{Wl^2}{16} (\frac{l}{2}) = \frac{-Wl^3}{48}$$

$$Y_c = \frac{-Wl^3}{48EI}$$

Cantilever with a uniformly distributed load

Consider a cantilever AB of length l and carrying a uniformly distributed load of 'w' per unit length.



Area of B.M. diagram = $\frac{wl^2 \times l}{2} \times \frac{1}{3} = \frac{wl^3}{6}$

VIII
(a)

Distance between CG of Bending Moment diagram and B

$$\bar{x} = \frac{3l}{4}$$

According to Mohr's theorem I, change of slope between A and B' is

$$\theta_B = \frac{A}{EI} = \frac{wl^3}{6EI} \text{ radians}$$

According to Mohr's theorem II, intercept of the tangents on A and B' is

$$BB' = Y_B = \frac{A\bar{x}}{EI} = \frac{wl^4}{8EI}$$

VIII
(b)

Solution-Given

$$l = 3\text{m}$$

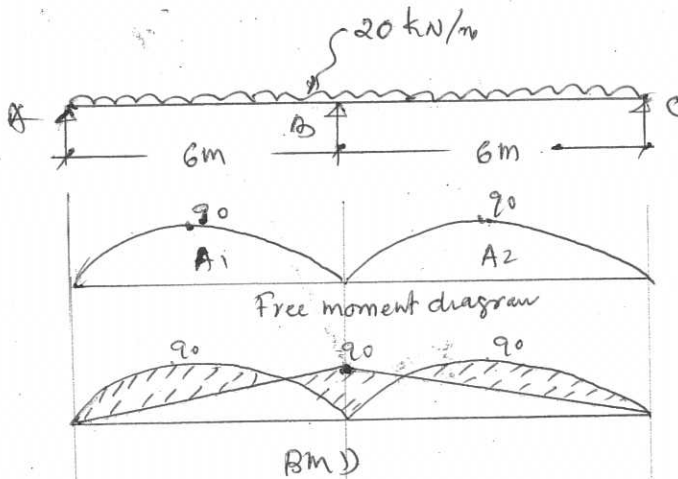
$$\text{Slope at ends} = 1^\circ = 0.01745 \text{ radian}$$

$$\text{Slope at the end, } \theta = \frac{wl^2}{16EI} = 0.01745 \text{ radian}$$

$$\begin{aligned} \text{Deflection at the centre, } Y_c &= \frac{wl^3}{48EI} = \frac{wl^2}{16EI} \times \frac{l}{3} \\ &= 0.01745 \times \frac{l}{3} = 0.01745 \times \frac{3}{3} \\ &= 1.745\text{cm} \end{aligned}$$

Y_c

IX
(a)



$$\text{Midspan ordinate of BMD} = \frac{wl^2}{8} = \frac{20 \times 6^2}{8} = 90 \text{ kNm}$$

∴ Area of moment diagram

$$\begin{aligned} \bar{x}_1 = \bar{x}_2 \text{ or } A_1 = A_2 &= \frac{2}{3} \times 6 \times 90 \\ &= 360 \text{ units} \end{aligned}$$

distance of centroid of A_1 from end A and A_2 from end C
 $a_1 = a_2 = 3$

ED in same through, $I_1 = I_2$

Clapeyron's theorem

$$M_A L_1 + 2M_B(L_1 + L_2) + M_C L_2 = \frac{-6A_1 \bar{x}_1}{L_1} + \frac{6A_2 \bar{x}_2}{L_2}$$

$$\text{i.e. } 6M_A + 24M_B + 6M_C = \frac{-6 \times 360 \times 3}{6} - \frac{6 \times 360 \times 3}{6}$$

Since support A B C are simple supports

$$M_A = M_C = 0$$

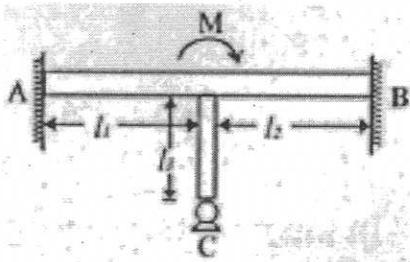
$$\therefore 24M_B = \frac{6 \times 360 \times 3}{6} - \frac{6 \times 360 \times 3}{6}$$

$$M_B = -90 \text{ kN-m}$$

$$= 90 \text{ kN m (hogging)}$$

IX
(b)

Consider three members OA, OB and OC meet at a rigid joint O. Let the ends A and B be fixed and let the end C be hinged.



Let M be the moment applied at joint O. Since O is rigid joint, all the members rotate by same angle, say θ . Let M_1 , M_2 and M_3 moments shared by members OA, OB and OC.

$$M_1 + M_2 + M_3 = M$$

Let k_1, k_2 and k_3 be the stiffness and l_1, l_2 and l_3 be the length of members

$$\begin{aligned} \theta &= M_1/k_1 = M_2/k_2 = M_3/k_3 \\ &= \frac{M}{\sum k} \end{aligned}$$

$$\therefore M_i = k_i \frac{M}{\sum k_i}$$

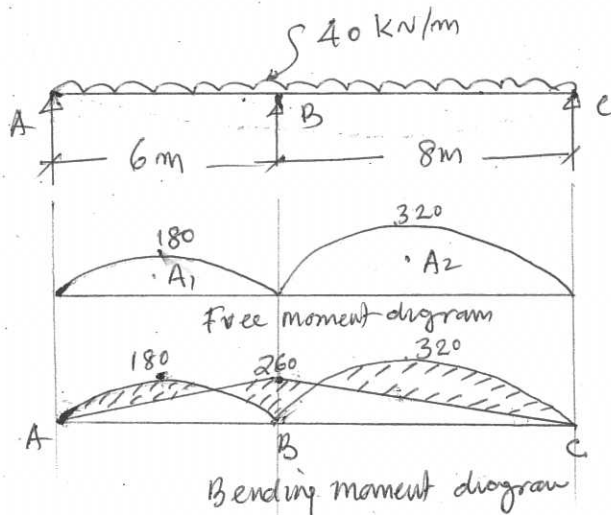
$$= \frac{k_i}{\sum k_i} M$$

Distribution factor for OA, $DF_{OA} = \frac{k_{OA}}{\sum k_O}$

Distribution factor for OB, $DF_{OB} = \frac{k_{OB}}{\sum k_O}$

Distribution factor for OC, $DF_{OC} = \frac{k_{OC}}{\sum k_O}$

X
(a)



Mid span moment in AB

$$= \frac{w l^2}{8} = \frac{40 \times 6^2}{8} = 180 \text{ kNm}$$

mid span moment in BC = $\frac{w l^2}{8} = \frac{40 \times 8^2}{8} = 320 \text{ kNm}$

$\therefore A_1 =$ Area of free BM diagram in span AB

$$A_1 = \frac{2}{3} \times 6 \times 180 = 720 \text{ units}$$

$$\bar{x}_1 = 3 \text{ m}$$

$A_2 =$ Area of free BM diagram in span BC

$$A_2 = \frac{2}{3} \times 8 \times 320 = 1706.67 \text{ units}$$

$$\bar{x}_2 = 4 \text{ m}$$

Flexural rigidity EI is constant

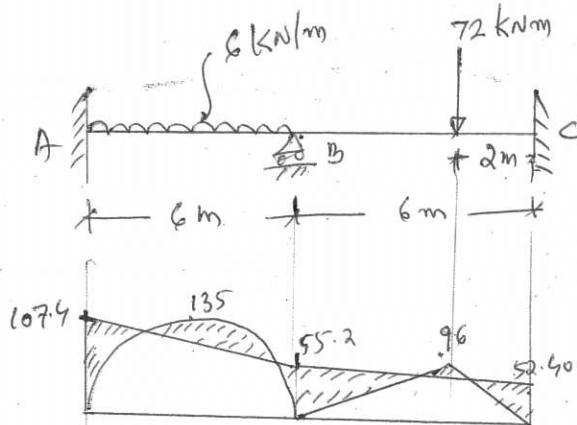
$$M_A \times 6 + 2M_B(6+8) + M_C \times 8 = -\frac{6 \times 720 \times 3}{6} - \frac{6 \times 1706.67 \times 4}{8}$$

Since ends A and C are simply supported, $M_A = M_C = 0$

$$\therefore 2 \times 14 M_B = - \frac{6 \times 720 \times 3}{6} - \frac{6 \times 1706.67 \times 6}{8}$$

$$\therefore M_B = -260 \text{ kNm (hogging)}$$

X
(b)



Fixed end moments

$$M_{FAB} = - \frac{30 \times 6^2}{12} = -90 \text{ kNm}$$

$$M_{FBA} = 90 \text{ kNm}$$

$$M_{FBC} = - \frac{72 \times 4 \times 2^2}{6^2} = -32 \text{ kNm}$$

$$M_{FCB} = \frac{72 \times 4 \times 3}{6^2} = 64 \text{ kNm}$$

Distribution factors

$$k_{BA} = \frac{4E(3I)}{6} = 2EI = \frac{2}{3.33} = 0.6$$

$$k_{BC} = \frac{4E(2I)}{6} = 1.33EI = \frac{1.33}{3.33} = 0.40$$

$$\Sigma k = 3.33EI$$

A	B		C
	0.6	0.40	
-90	90	-32	64
-17.40 ←	-34.80	-23.20	→ -11.60
-107.40	55.20	-55.20	52.40

Qst.no	Scoring indicator	Split up score	Sub total	total
	<p>For span AB, free Bm diagram = $\frac{wL^2}{8}$</p> $= \frac{30 \times 6^2}{8} = 135 \text{ kNm}$ <p>For span BC; free Bm diagram = $\frac{Wab}{l}$</p> $= \frac{72 \times 4 \times 2}{6} = 96 \text{ kNm}$ <hr style="width: 20%; margin-left: auto; margin-right: auto;"/>	<p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p>		7