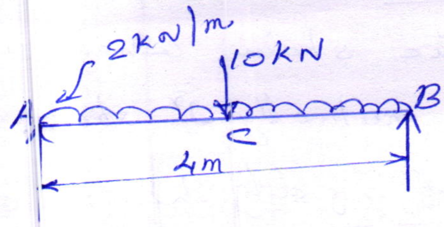
(15) 3013

DE 2017

Scoring Indicators

Code : 3013 - Theory of Structures-I **Version:** B

Qn. No.	Scoring Indicators	Split score	Total score
	Part A		
I. 1.	Magnitude, direction, nature of force, the point at which the force acts on the body.	$4 \times \frac{1}{2} = 2$	
3.	The ratio of lateral strain to linear strain will be a constant within elastic limit. This ratio is known as Poisson's ratio.	2	
2.	The $\perp^r$ distance at which the whole mass or area of the body is concentrated to the reference axis.	2	
4.	It is the turning moment or twisting moment of a force. It is the product of force and the distance from the point of application of force and the centre of shaft	2.	
5.	Moment of resistance is the moment of the couple formed by the compressive force and tensile force due to bending or external bending moment.	2	10
	Part B		
II 1.			
	$R_B \times 4 = 2 \times 4 \times 2 + 10 \times 2$	moment eqn - 2	
	$\therefore R_B = 9\,kN$	R_A - 2	
	$\therefore R_A = 18 - 9 = \underline{9\,kN}$.	R_B - 2	
		6	

Scoring Indicators

Code : **Version:**

Qn. No.	Scoring Indicators	Split score	Total score
2.	Sphere $V_1 = \frac{2\pi r^3}{3} = \frac{2\pi \times 30^3}{3} = 18000\pi\ mm^3$. $y_1 = 40 + \frac{3}{8}r = 40 + \frac{3\times 30}{8} = 51.25\ mm$. Cone $V_2 = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi \times 30^2 \times 40 = 12000\pi\ mm^3$. $y_2 = \cancel{\frac{3\times 4}{4}} = \cancel{10\ mm}\ \frac{3}{4}\times 40 = 30\ mm$. $\therefore \bar{y} = \frac{V_1 y_1 + V_2 y_2}{V_1 + V_2} = \frac{18000\pi \times 51.25 + 12000\pi \times 30}{30000\pi}$ $= 42.75\ mm$ from the top C.	eqn - 2 y_1, y_2 - 2 $\bar{y}$ - 2 6	
3.	$P = 20\ kN,\ l = 1\ m,\ A = 200\ mm^2,\ E = 100\ GPa$ $\delta l = \frac{Pl}{AE} = \frac{20\times 10^3 \times 1000}{200 \times 100 \times 10^3} = 1\ mm$	eqn - 2 $\delta l = 4$ 6	
4.	Volumetric strain: When the body is subjected to some forces, the ratio of change in volume to the original volume is called volumetric strain. Bulk modulus: The ratio of direct stress to volumetric strain, when a body is subjected to three mutually ⊥r stresses of equal intensity. Modulus of rigidity: Within elastic limit, the ratio of shear stress to shear strain is a constant. This constant is known as modulus of rigidity.	2 2 2 6	

1	2	

5. $T_{ave} = 2255\,Nm$, $D = 80\,mm$.

Max. Torque = $1.4 \times 2255 = 3157\,Nm$.

$$T = \frac{\pi}{16} \sigma_s D^3$$ eqn - 2

$$\therefore \sigma_s = \frac{16T}{\pi D^3} = \frac{16 \times 3157 \times 10^3}{\pi \times 80^3} = \underline{\underline{31.4\,N/mm^2}}$$ σ_s - 4

6

6. Longitudinal stress:- It is the stress induced in the material of the thin cylinder in a direction parallel to the length of the shell.

$\sigma_l = \frac{Pd}{4t}$, where P = pressure of internal fluid
d = diameter of the cylinder.
t = thickness of the cylinder.

when this stress exceeds the permissible tensile stress of material of cylinder, the cylinder will burst into two cylinders. 3

Hoop or circumferential stress:- It is the tensile stress induced in the material of thin cylinder containing some fluid under pressure, in a direction tangential to the perimeter of the cylinder. If the stress exceeds the permissible stress, the cylinder will burst into two halves. 3

6

7. 1) The material of the beam is homogeneous and isotropic

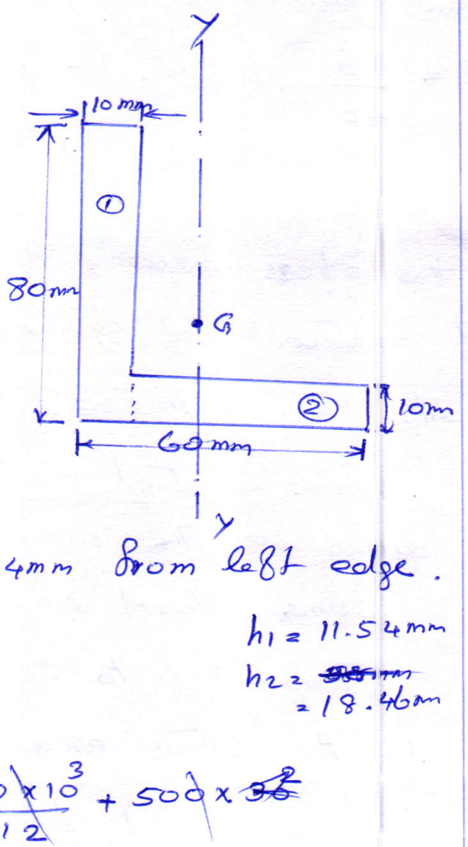
2) The value of young's modulus of elasticity is same in tension and compression.

3) The transverse sections which were plane

2	3	4
remains plane after bending. initially straight and all longitudinal ...nd into circular arcs with a ...te of curvature.		
5) The radius of curvature is large compared with the dimensions of cross section.		
6) Each layer of the beam is free to expand or contract, independently of the layer above or below it.	$6 \times 1 = 6$	42

Part C

III (a) $a_1 = 80 \times 10 = 800\, mm^2$



$x_1 = 5\, mm$

$a_2 = 50 \times 10 = 500\, mm^2$

$x_2 = 35\, mm$

$$\bar{x} = \frac{a_1 x_1 + a_2 x_2}{a_1 + a_2}$$

$$= \frac{800 \times 5 + 500 \times 35}{1300} = \underline{16.54\, mm} \text{ from left edge.}$$

$h_1 = 11.54\, mm$

$h_2 = \cancel{\ldots} = 18.46\, mm$

$$I_{yy} = I_{G1} + a_1 h_1^2 + I_{G2} + a_2 h_2^2$$

$$= \cancel{\frac{10 \times 80^3}{12}} + 800 \times 11.54^2 + \cancel{\frac{50 \times 10^3}{12}} + 500 \times \cancel{30}^2$$

$$= \frac{80 \times 10^3}{12} + 800 \times 11.54^2 + \frac{10 \times 50^3}{12} + 500 \times 18.46^2$$

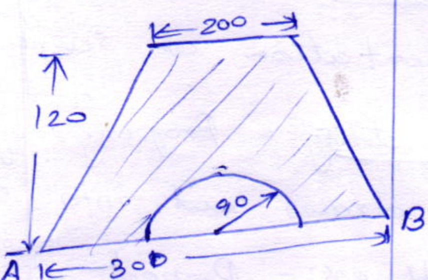
$$= 113203.95 + 274552.47$$

$$= \underline{\underline{387756.42\, mm^4}}$$

$\bar{x} = 3$
eqn - 2
I_{yy} - 3
8

1	2	3	4.

b) $a_1 = \frac{(200+300)}{2} 120 = 30000 \; mm^2$



$y_1 = \frac{h}{3}\left(\frac{b+2a}{b+a}\right) = \frac{120}{3}\left[\frac{300+2\times200}{300+200}\right]$

$= 56 mm.$

$a_2 = \frac{1}{2}\pi r^2 = \frac{1}{2}\pi \times 90^2 = 12723.45 \; mm^2$

$y_2 = \frac{4r}{3\pi} = \frac{4\times90}{3\pi} = 38.2 mm.$ eqn-2

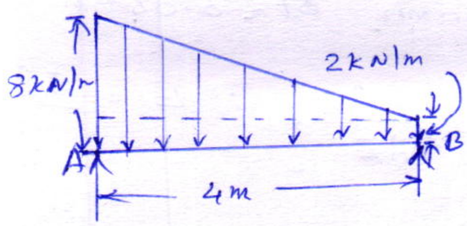
$\bar{y} = \frac{a_1 y_1 - a_2 y_2}{a_1 - a_2} = \frac{30000\times56 - 12723.45\times38.2}{30000 - 12723.45}$ $\bar{y}$ - 5

7 15

$= \underline{69.1 mm}$ from AB. , $\bar{x} = 150$ mm from A

IV(a)

$R_A + R_B = \frac{(8+2)}{2} 4 = 20 kN$



$R_B \times 4 = 2\times4\times2 + \frac{1}{2}\times8\times4\times\frac{4}{3}$ moment-4

$R_B = \underline{9.33 kN}$ R_A -2

$R_A = 20 - 9.33 = \underline{10.67 kN}$ R_B - 2

8

(b) $I_{zz} = I_{xx} + I_{yy}$ $b = 200 mm, d = 300 mm.$

$= \frac{bd^3}{12} + \frac{db^3}{12}$ I_{xx} - 2

I_{yy} - 2

$= \frac{200\times300^3}{12} + \frac{300\times200^3}{12}$ I_{zz} - 3

7 15

$= 450\times10^6 + 200\times10^6 = \underline{650\times10^6 \; mm^4}$

V (a)(i) Elasticity: It is the property of a material by which the ~~load~~ the deformation due to an external load is disappear on removal of the load. 2

ii) Hardness :- It is the property of a material to resist indentation or surface abrasion — 2

iii) Ductility :- Property by which the material can be drawn out into thin wires. — 2

iv) Stiffness :- Property of resistance to bending — 2

8

b) $\sigma = 76.5\ Mpa$, $E = 90\ GPa$, $\alpha = 17\times10^{-6}/^\circ C$

$$\sigma = \alpha t E$$

$$\therefore t = \frac{\sigma}{\alpha E} = \frac{76.5\times10^{6}}{17\times90\times10^{3}} = 50^\circ C$$

eqn - 2

t - 5

7 — 15

VI(a) $A = 50\times50 = 2500\ mm^2$, $P = 500\ kN$, $\delta l = 0.5\ mm$,

$l = 200\ mm$, $\delta t = 0.04\ mm$

$$\delta l = \frac{Pl}{AE}, \quad \therefore E = \cancel{\delta l \cdot A}\ \frac{Pl}{A\cdot\delta l}$$

$$= \frac{500\times10^3\times200}{2500\times0.5} = 80\times10^3\ N/mm^2$$

E = 4

$\frac{1}{m}$ = 4

8

$$\frac{1}{m} = \frac{\delta t/t}{\delta l/l} = \frac{0.04/\cancel{50}}{0.5/200} = 0.32$$

b) $A = 500\times200 = 100000\ mm^2$, $l = 2m$, $P = 150\ kN$, ~~E = 200~~ $E = 200\ kN/mm^2$.

stress at elastic limit $= 200\ N/mm^2$.

i) Strain energy $= \frac{\sigma^2}{2E}\times V = \frac{\left(\frac{150}{10^5}\right)^2}{2\times200}\times10^5\times2000 = 1.125\ kNmm$ — 2

ii) Proof resilience $= \frac{\sigma^2}{2E}\times V = \frac{0.2^2}{2\times200}\times10^5\times2000 = 20000\ kNmm.$ — 3

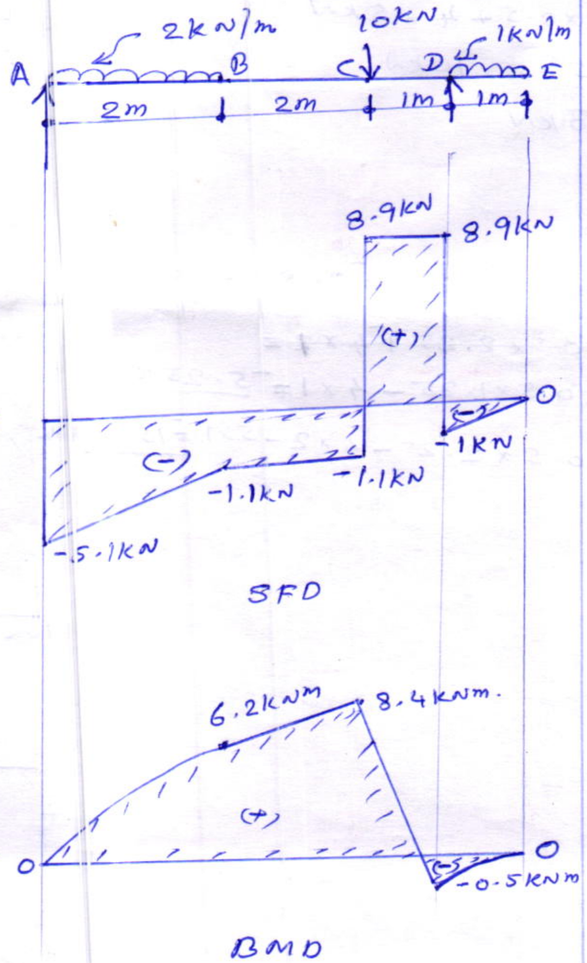
iii) Modulus of resilience $= \frac{\text{Proof resilience}}{V} = 0.1\ N/mm^2$ — 2

7 — 15

1	2	3	4

VII(a) $R_D \times 5 = 1\times1\times5.5 + 10\times4 + 2\times2\times1$

$\therefore R_D = \frac{49.5}{5} = \underline{9.9kN}$



$R_A + R_D = 15$

$\therefore R_A = 15 - 9.9 = \underline{5.1 kN}$

SF

at $E = 0$

" $D = 0 - 1 = -1kN + 9.9 = \underline{8.9kN}$

" $C = 8.9 - 10 = \underline{-1.1kN}$

" $B = -1.1kN$

" $A = -1.1 - 4 = -5.1kN$

BM

at $E = \underline{0}$.

" $D = -1\times1\times0.5 = \underline{-0.5kNm}$.

" $C = -1\times1\times1.5 + 9.9\times1$

$= \underline{8.4kNm}$.

" $B = 5.1\times2 - 2\times2\times1$

$= \underline{6.2kNm}$

" $A = 0$.

Max +ve BM $= \underline{8.4kNm}$.

Max −ve BM $= \underline{-0.5kNm}$.

SFD 4
BMD-4
8

b) $D = 80$ mm, $\theta = 1.5^\circ$, $l = 5$ m, $\sigma_s = 42$ MPa, $N = 84$ GPa

$T = \frac{\pi}{16}\sigma_s D^3 = \frac{\pi}{16}\times 42\times 80^3 = \underline{4222.3\times10^3}$ Nmm. — 3

T is also equal to $\frac{N\theta\times J}{l}$, $J = \frac{\pi D^4}{32} = \frac{\pi\times80^4}{32} = 50265.48$

$\therefore T = \frac{84\times10^3\times50265.48\times\frac{\pi}{180}\times1.5}{5000} = \underline{1768.63\times10^3}$ Nmm — 3

$\therefore$ Maximum torque that can be applied = lesser value

$= 1768.63\times10^3$ Nmm $= \underline{\underline{1768.63 Nm}}$ — 1

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VIII (a) SF

at D = 0

" C = 2×0.5 + 4 = 5 kN

" B = 5 + 5 = 10 kN

" A = 10 kN

BM

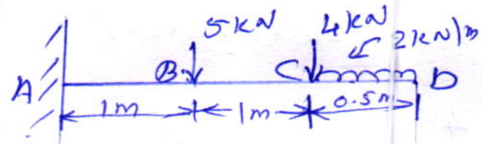
at D = 0

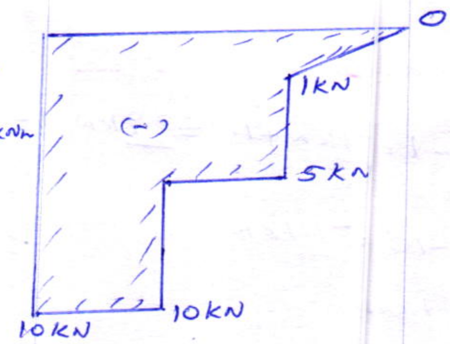
" C = 2×0.5×0.25 = ‾0.25 kNm

~~" B = 2×0.5×2.25 + 4×1 =~~

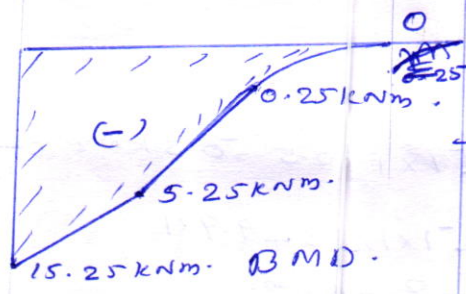
at B = ‾2×0.5×1.25 − 4×1 = ‾5.25 kNm

" A = ‾2×0.5×2.25 − 4×2 − 5×1 = ‾15.25 kNm





SFD — 4



BMD — 4

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b) $d = 2m$, $t = 10mm$, $P = 1.6\ MPa$, $E = 200\ GPa$, $\frac{1}{m} = 0.3$

Change in diameter, $\delta d = \frac{Pd^2}{4tE}\left[1 - \frac{1}{m}\right] = \frac{1.6 \times 2000^2}{4 \times 10 \times 200 \times 10^3}\left[1 - 0.3\right]$

$= \underline{0.56\ mm}$ — 3

Change in volume, $\delta V = \frac{\pi P d^4}{8tE}\left[1 - \frac{1}{m}\right]$

$= \frac{\pi \times 1.6 \times 2000^4}{8 \times 10 \times 200 \times 10^3}(1 - 0.3)$

$= \underline{3518.583 \times 10^3\ mm^3}$ — 4

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IX (a) $b = 200\ mm$, $d = 400\ mm$, $w = 10\ kN/m$, $\sigma_{max} = 50\ N/mm^2$

$M = \frac{wl^2}{8} = \frac{10 \times l^2}{8} = 1.25\, l^2$

$$I = \frac{bd^3}{12} = \frac{200\times400^3}{12} = 1066.67\times10^6\ mm^4$$

$$Z = \frac{I}{y_{max}} = \frac{1066.67\times10^6}{200} = 5.333\times10^6\ mm^3.$$

$$M = \sigma_{max}\times Z = \frac{\omega l^2}{8} = 1.25 l^2$$

$$\therefore l^2 = \frac{50\times5.333\times10^6}{1.25\times10^6} = 213.3$$

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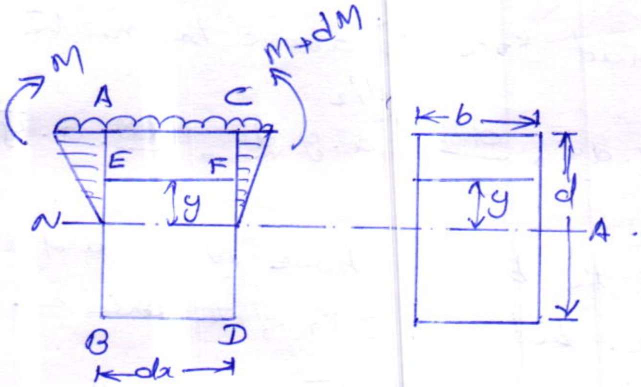
$$\therefore l = 14.6m$$

b)



Consider a small part of beam ABCD of length dx is loaded with a u.d.l as shown in fig.

Let M = BM at Section AB
$M+dM$ = BM at section CD
F = Shear force at AB,
$F+dF$ = shear force at CD
I = Moment of Inertia of the Section about its N.A.

consider an elementary strip at a distance 'y' from N.A.

Let σ = intensity of bending stress across AB at a distance y from N.A.

a = area of strip

$$\frac{M}{I} = \frac{\sigma}{y} \quad \text{or} \quad \sigma = \frac{M.y}{I}$$

$$\therefore \sigma + d\sigma = (M + dM) \times \frac{y}{I}$$

$\therefore$ the force across AB = stress × area = $\sigma \times a$

$$= \frac{M}{I} \times y \times a.$$

~~and the force across~~

the force across CD = $(\sigma + d\sigma)\, a = \frac{(M + dM)}{I} \times y \times a.$

$\therefore$ Net unbalanced force = $\frac{(M + dM)}{I} \times y \times a - \frac{M}{I} \times y \times a$

$$= \frac{dM}{I} \times y \times a.$$

The total unbalanced force (F) above the neutral axis = $\int_0^{d/2} \frac{dM}{I} . a . y . dy = \frac{dM}{I} \int_0^{d/2} a . y . dy = \frac{dM}{I} . A . \bar{y}.$

where, A = area of the beam above N.A and

$\bar{y}$ = distance between C.G of the area and N.A.

We know that $\tau = \frac{Force}{Area} = \frac{\frac{dM}{I} . A\bar{y}}{dx . b} = \frac{dM}{dx} . \frac{A\bar{y}}{Ib}$

$$= \frac{F . A\bar{y}}{Ib} \qquad \left(\because \frac{dM}{dx} = F \right)$$

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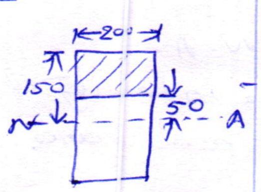
x (b) b = 200mm, d = 300mm, F = 100kN.

$$\tau_{max} = 1.5\, \tau_{av} = 1.5 \times \frac{F}{bd} = \frac{1.5 \times 100 \times 10^3}{200 \times 300} = \underline{\underline{2.5\ N/mm^2}} \quad \text{--- } 3$$

τ at y = 50mm, $\tau = \frac{F . A\bar{y}}{Ib} = \frac{100 \times 10^3 \times 200 \times 100 \times 100}{\frac{200 \times 300^3}{12} \times 200}$

$$= \underline{\underline{2.22\ N/mm^2}}$$



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